

Estimation bias of mineral deposits caused by grade-based staging of multiple analyses in samples exhibiting a 'nugget effect'

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Good Morning.

What we would like to do today is make geochemists aware of a potential pitfall associated with assay strategies designed to address sampling problems associated with deposits exhibiting the 'nugget effect'.

This pitfall can occur in exploration projects involving a variety of deposit types, including those containing Au, Pt-Pd, diamonds, and others with rare grains (or nuggets) of economic interest.

This pitfall has previously gone un-recognized, but can lead to significant grade estimation biases that may undermine the economic feasibility of a mining venture.

Problem

- exploration/resource assessment datasets commonly contain replicate assays for elements affected by the nugget effect (*typically undertaken to avoid nugget effect and obtain better estimates of grade*)
- assay used is typically the mean of all replicate values
- sometimes, the decision to undertake replicate analysis is based on the initial grade obtained in analysis
- the fact that samples with different initial grades are treated differently results in a bias
- *this bias can sometimes be very significant*
- *bias is unpredictable without a priori knowledge of size & # of nuggets in the samples (=> un-correctable)*
- *possible important/catastrophic consequences!*

One strategy for avoiding or minimizing the impact of the 'nugget effect' on mineral exploration and resource assessments involves undertaking replicate analysis of the samples. Once averaged, these replicate analyses effectively increase the 'size' of the sample analyzed, and thus reduce the variation associated with the random inclusion or omission of coarse nuggets.

However, historically mining companies have commonly employed analytical regimens that involve an initial sample assay, the magnitude of which guides subsequent analysis. For example, if the assay reports an initial high grade, subsequent replicate assays may be undertaken and averaged to obtain a better estimate of the true grade; however, if the assay reports an initial low grade, no additional assays are made.

Unfortunately, because samples are treated differently depending on grade, a bias is introduced. This bias can be significant, and is not predictable unless a priori knowledge of the sizes and number of nuggets in the samples is available. As a result, this bias can have important or even catastrophic consequences.

Staged Replicate Sampling Scheme

- 500 gram pulverized sample
- 30 gram sub-sample with an initial grade
(470 grams left over)
- re-assay criteria:
 - if grade < 2 gpt ⇒ use result as grade
 - if grade is between 2 & 5 gpt ⇒ take 2nd 30 gram split and assay again; use average of two as grade
 - if grade > 5 gpt ⇒ take 3 additional 30 gram splits; assay them; use average of four as grade
- **Results in Biased Estimate of Grade!!!**

To illustrate this bias, consider a scenario where a 30 gram sub-sample is split from a 500 gram pulverized sample and analyzed for gold. Based on the result of this first assay, additional assays may be undertaken.

- If the grade < 2 gpt, the sub-sample did not likely contain large nuggets.
- If the grade > 2 & < 5 gpt, the sub-sample may have contained nuggets, and a second 30 gram sub-sample collected from the original 500 gram (now 470 gram) pulverized sample would, after assaying and averaging, provide a better grade estimate.
- However, if the grade > 5 gpt, then nuggets most certainly existed in the sub-sample, and three additional 30 gram sub-samples from the original pulverized sample might be necessary to, after analysis and averaging, provide an adequate grade estimate.

Unfortunately, this approach does not recognize that any initial low grade assays may be derived from samples containing large nuggets (because the nuggets were randomly omitted), and thus the first analysis cannot be used reliably to guide subsequent analytical procedures designed to avoid or mitigate the 'nugget effect'.

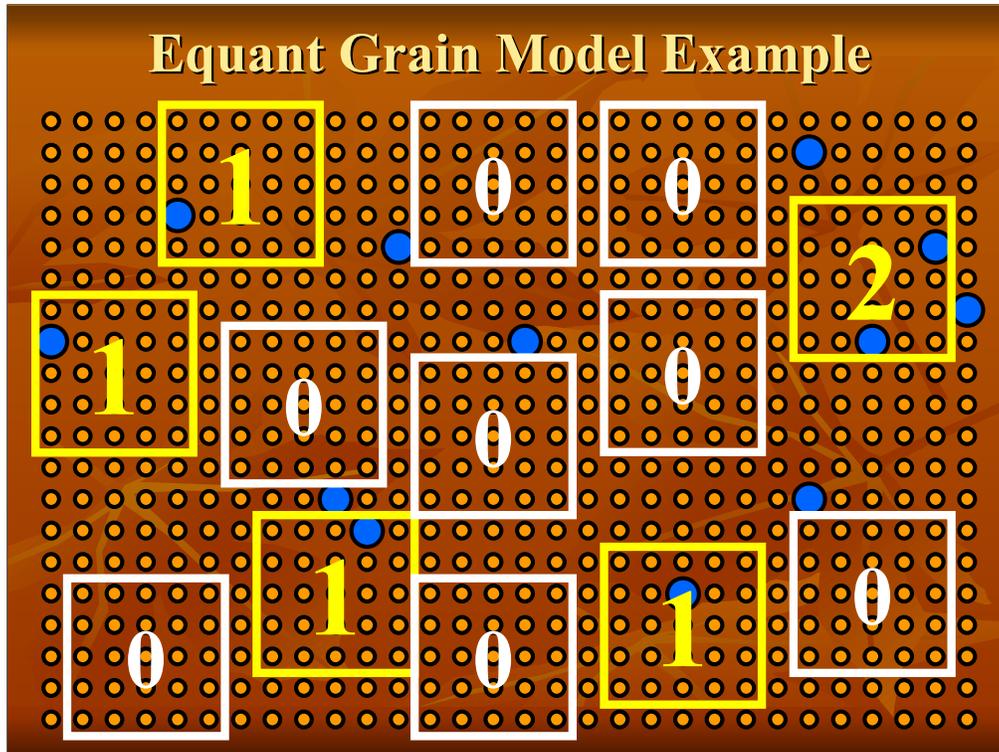
Equant Grain Model

- **Equant Grain Model**
 - samples composed of equal sized, spherical grains
 - element of interest occurs in one grain type (*the nugget*)
 - small number of nuggets occur in sample (< 50)
 - large number of 'gangue' grains occur in sample (> 10,000)
- **Allows use of Poisson distribution to determine how many nuggets occur per sample**

To illustrate the magnitude of this bias, a simple model must be used. We have invoked the 'Equant Grain Model', which assumes that all (nugget and gangue) grains are the same size and shape (in this case, spherical) to illustrate this problem.

This model assumes that the element of interest occurs in only one mineral or grain type, that there are a small number of nuggets, and that there are a large number of gangue grains.

If true, the number of nuggets contained within a sample or sub-sample will be governed by the Poisson distribution.



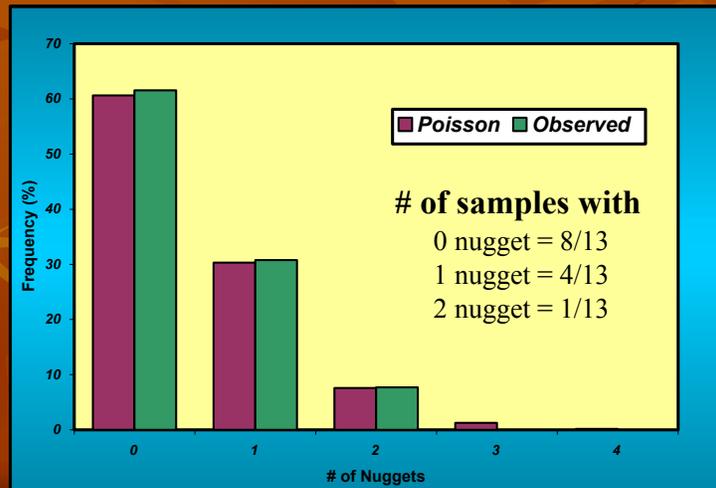
An example of this model is presented here, with a number of sub-samples from a larger sample.

In each of these sub-samples (the squares), a different number of nuggets (in blue) are obtained.

As a result, the grades of these sub-samples vary, and the frequency distribution describing the number of grains obtained in the sub-samples is the Poisson distribution.

Equant Grain Model Example

- 600 grains total
- sub-samples of 25 grains
- 12 nuggets
- average = $\frac{1}{2}$ nugget/sub-sample



Here, we compare a histogram of the results from the previous slide with the Poisson distribution having the same mean number of nuggets per sub-sample (= $\frac{1}{2}$ nuggets/per sample).

This admittedly 'cooked' example illustrates how the Poisson distribution can be used to describe geochemical variation associated with the 'nugget effect'.

Simulation to Assess Bias in Staged Replicate Sampling Scheme

- **baseline** => 3 nuggets per 30 gram split average
70 um diameter spherical nugget
assumed pure gold nugget ($\rho = 19.3 \text{ g/ml}$)
 1.05×10^8 grains total
1.04 gpt grade
- Modified the number of nuggets/sample (0.3 <=> 4.5)
- Modified the size of grains (10 <=> 150 um)
- Resulted in total number of grains between
 1.03×10^8 <=> 3.59×10^{11}
- Resulted in gold grades between 0.001 and 15.348 gpt

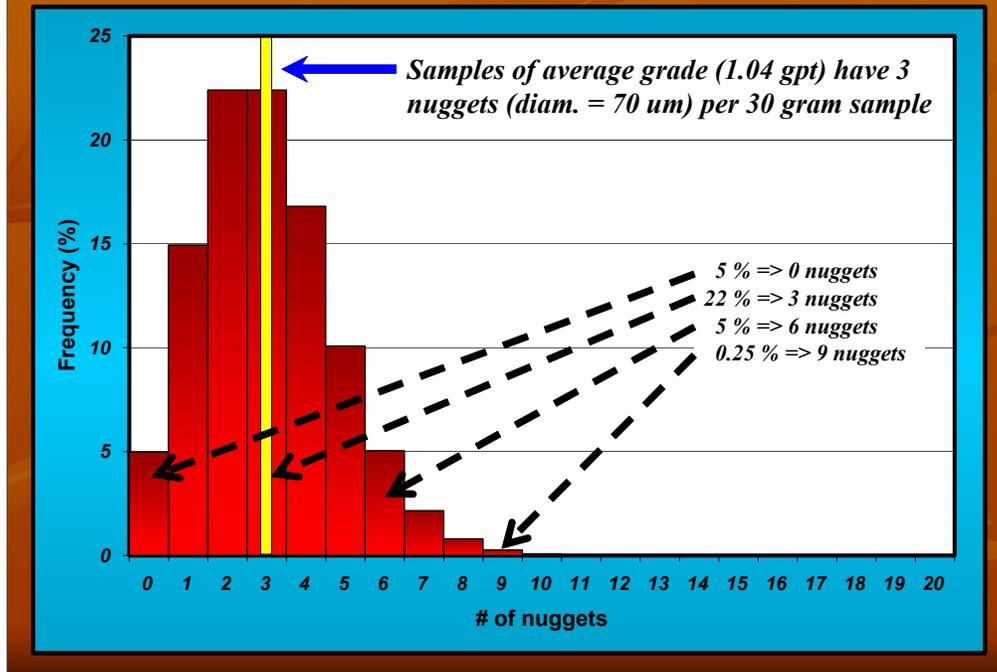
Because the 'Equant Grain Model' and Poisson statistics can be used to describe the 'nugget effect', we have used these in a mathematical simulation to determine the magnitude of the bias associated with a representative grade-dependent sampling and analysis scheme.

We have chosen a baseline case where we have 3 gold nuggets per sub-sample (on average), and 70 um diameter spherical nugget and gangue grains. We also assume the gold is pure, and that gold does not occur in the gangue grains.

With these assumptions, there will be approximately 100 million grains per 500 gram pulverized sample, and the Au concentration will be 1.04 gpt.

In our simulation, we also varied the number of nuggets per sub-sample (from 0.3 to 4.5) and the size of the grain diameters (from 10 to 150 um). This resulted in a range in the total number of grains per pulverized sample, from less than 100 million to over 300 billion (satisfying the assumptions required for invoking of the Poisson distribution), and a range in gold grades from < 0.001 gpt to > 15 gpt.

Simulation Baseline



This histogram illustrates the expected number of nuggets we would obtain by sub-sampling our baseline case a number of times. The resulting gold grades vary substantially, due to the random inclusion of a different number of nuggets in each sub-sample.

The true # of nuggets is obtained only 22 % of the time; grades less than the true # of nuggets are obtained 42 % of the time, and grades greater than the true # of nuggets are obtained 36 % of the time.

Clearly, using the first analysis as a basis to determine how to further analyze a sample can result in significant errors in subsequent sample analysis, as incorrect grades are obtained initially more than $\frac{3}{4}$ of the time.

Perhaps I should stop there!

Staged Replicate Sampling Scheme

< 2 gpt => 1 analysis
 2 to 5 gpt => 2 analyses
 > 5 gpt => 4 analyses

baseline

Expected Grade (gpt)		Nugget Size (um)														
		10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
	5	<0.1	<0.1	<0.1	<0.1	<0.1	0.1	0.1	0.2	0.2	0.3	0.4	0.5	0.7	0.8	1.0
	10	<0.1	<0.1	<0.1	<0.1	0.1	0.1	0.2	0.3	0.4	0.6	0.8	1.0	1.3	1.7	2.0
	15	<0.1	<0.1	<0.1	0.1	0.1	0.2	0.3	0.5	0.7	0.9	1.2	1.6	2.0	2.5	3.1
	20	<0.1	<0.1	<0.1	0.1	0.2	0.3	0.4	0.6	0.7	1.0	1.2	1.6	2.1	2.7	3.3
#	25	<0.1	<0.1	<0.1	0.1	0.2	0.3	0.5	0.8	1.1	1.5	2.0	2.6	3.3	4.2	5.1
of	30	<0.1	<0.1	<0.1	0.1	0.2	0.4	0.6	0.9	1.3	1.8	2.4	3.1	4.0	5.0	6.1
Nuggets	35	<0.1	<0.1	0.1	0.1	0.3	0.5	0.7	1.1	1.5	2.1	2.8	3.7	4.7	5.8	7.2
per	40	<0.1	<0.1	0.1	0.2	0.3	0.5	0.8	1.2	1.8	2.4	3.2	4.2	5.3	6.7	8.2
500 g	45	<0.1	<0.1	0.1	0.2	0.3	0.6	0.9	1.4	2.0	2.7	3.6	4.7	6.0	7.5	9.2
Sample	50	<0.1	<0.1	0.1	0.2	0.4	0.7	1.0	1.6	2.2	3.0	4.0	5.2	6.7	8.3	10.2
	55	<0.1	<0.1	0.1	0.2	0.4	0.7	1.1	1.7	2.4	3.3	4.4	5.8	7.3	9.2	11.3
	60	<0.1	<0.1	0.1	0.2	0.5	0.8	1.2	1.9	2.7	3.6	4.8	6.3	8.0	10.0	12.3
	65	<0.1	<0.1	0.1	0.3	0.5	0.9	1.4	2.0	2.9	3.9	5.2	6.8	8.7	10.8	13.3
	70	<0.1	<0.1	0.1	0.3	0.5	0.9	1.5	2.2	3.1	4.2	5.6	7.3	9.3	11.6	14.3
	75	<0.1	<0.1	0.1	0.3	0.6	1.0	1.6	2.3	3.3	4.5	6.1	7.9	10.0	12.5	15.3

This table presents the various nugget simulations we considered.

On the left of this matrix is the number of nuggets per 500 gram pulverized sample that we considered; on the top of this matrix is the diameters of the nuggets we considered.

Gold grades increase from left to right and top to bottom on this chart, as the nugget size (ranging from 10 to 150 um) and number of nuggets per sample (ranging from 5 to 75) increase. Highest grades occur in the brownest boxes in the lower right corner of the matrix. This corresponds to the case with the largest number of largest nuggets in the sample.

Our baseline case is identified in the box. It has a grade of 1.04 gpt with 70 um diameter spherical gold nuggets, and 50 nuggets in the pulverized sample). It is used in the following slide to illustrate the simulation calculations.

Simulation to Assess Bias in Staged Replicate Sampling Scheme

# nuggets in first split =	0	3	6	9	12	15	18
# nuggets in initial sample =	50	50	50	50	50	50	50
# nuggets in first 30 g sub-sample =	0	3	6	9	12	15	18
# nuggets in left-over material =	50	47	44	41	38	35	32
initial average # nuggets in 30 grams =	3.000	3.000	3.000	3.000	3.000	3.000	3.000
subsequent average # of nuggets in 30 grams =	3.191	3.000	2.809	2.617	2.426	2.234	2.043
# of followup 30 gram analyses =	0	0	1	1	1	3	3
total # of analyses =	1	1	2	2	2	4	4
Poisson Probability (u = 3.0; in %) =	4.98	22.40	5.04	0.27	0.01	0.00	0.00
initial sample grade (gpt) =	1.040	1.040	1.040	1.040	1.040	1.040	1.040
first sub-sample grade (gpt) =	0.000	1.040	2.080	3.120	4.159	5.199	6.239
average left-over sample grade (gpt) =	1.106	1.040	0.973	0.907	0.841	0.774	0.708
average of all replicates (gpt) =	0.000	1.040	1.527	2.013	2.500	1.881	2.091

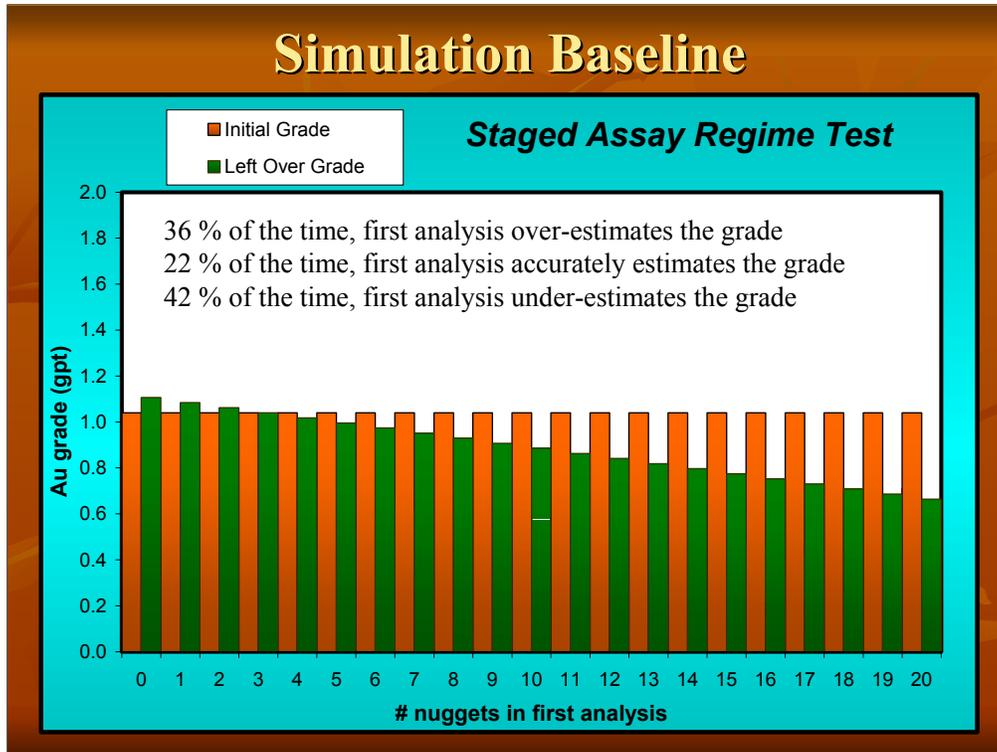
Although our baseline scenario has an average of 3 nuggets per 30 gram sub-sample, the actual first sub-sample may contain any number of nuggets (up to 50) with probabilities described by the Poisson distribution.

As a result, we must simulate each of these possible scenarios (where the initial sub-sample had 0, 1, 2, 3 nuggets, etc.), by first determining how many nuggets occur in the left-over material.

Given the number of nuggets collected in the first sub-sample, we can calculate the initial grades, as well as the grades of the material left-over. The 1 or 3 subsequent assays of this left-over material will report this 'left-over' grade, on average. As a result, for each scenario with different #'s of nuggets in our first assay, we can calculate the expected average grade for all (either 1, 2 or 4) of our replicate analyses.

Finally, we can calculate the average grade expected via this sampling and analysis procedure, a weighted average calculation where the weights are the Poisson probabilities for each scenario.

Simulation Baseline

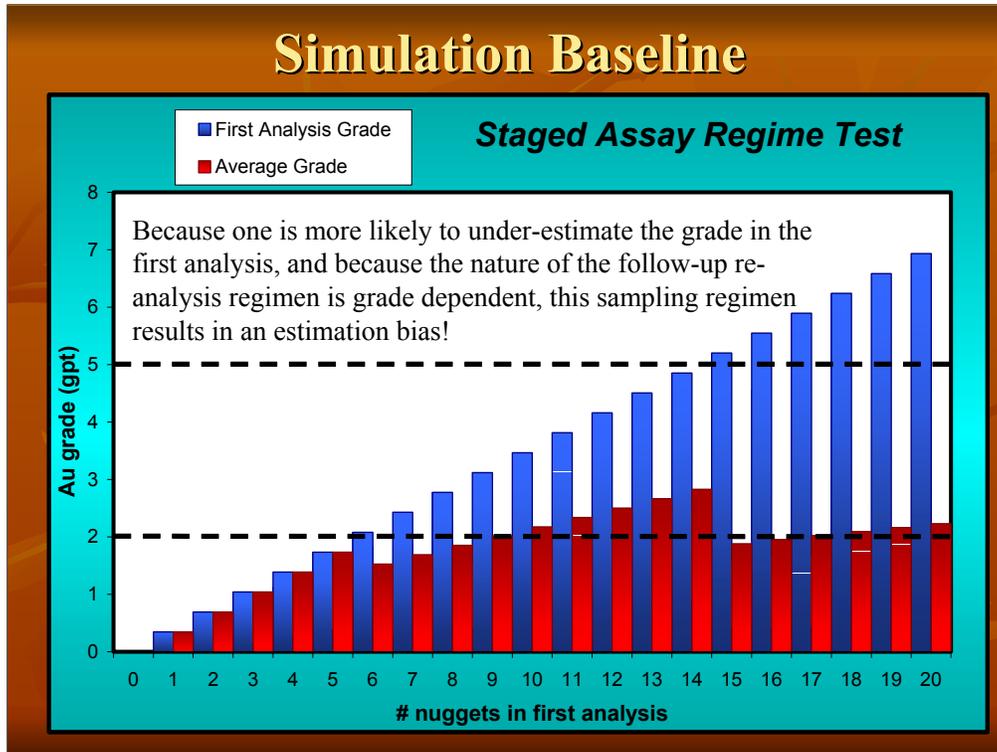


The true grade of 1.04 gpt is represented in orange for each scenario involving a first sub-sample containing a given number of nuggets.

The different scenario grades of the left-over material, after the first sub-samples were collected, are presented in green.

Clearly, the grade of the left-over material changes for each scenario, so if this material is not subsequently re-sub-sampled and analyzed in a routine and identical manner, differences in the average replicate grade can result.

Simulation Baseline



Obviously, the grade of the initial sub-sample (in blue) varies with the number of nuggets collected in that sub-sample. After using this initial grade to determine how many additional sub-samples (0, 1 or 3) are assayed and averaged, the expected average replicate grade can be determined (in red).

Note that this expected average replicate grade equals the initial sub-sample grade when the initial grade is less than 2 gpt, as expected, because no additional sub-samples were collected or analyzed. Furthermore, note that two discontinuities occur in the expected average replicate grade (in red) where the initial grade crosses the 2 and 5 gpt levels, precisely because these are the thresholds where subsequent sample treatment changes with grade.

Finally, because the initial sub-sample assays provide an average grade estimate that is un-biased, the fact that the expected average replicate grades for most scenarios differ from these initial sub-sample grades indicates that the expected average replicate grades are biased (in this case, downward).

Staged Replicate Sampling Scheme

< 2 gpt => 1 replicate
 2 to 5 gpt => 2 replicates
 > 5 gpt => 4 replicates

baseline

Estimation Bias (in % of true grade)		Nugget Size (um)														
		10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
	5	100	100	100	100	100	100	100	100	100	106	106	106	185	188	188
	10	100	100	100	100	100	100	100	100	100	108	108	108	167	171	171
	15	100	100	100	100	100	100	100	99	99	109	108	108	152	156	156
	20	100	100	100	100	100	100	100	98	98	107	107	107	139	144	144
#	25	100	100	100	100	100	100	99	97	97	105	104	104	128	132	132
of	30	100	100	100	100	100	100	99	95	95	102	101	101	118	122	122
Nuggets	35	100	100	100	100	100	100	98	94	93	99	97	97	110	114	114
per	40	100	100	100	100	100	100	97	91	91	95	94	94	103	106	106
500 g	45	100	100	100	100	100	100	96	90	89	91	90	90	97	99	99
Sample	50	100	100	100	100	100	100	95	88	87	88	86	86	91	93	93
	55	100	100	100	100	100	100	93	86	84	84	83	83	86	88	88
	60	100	100	100	100	100	100	91	84	82	81	79	79	82	83	83
	65	100	100	100	100	100	99	89	82	79	77	76	76	78	79	79
	70	100	100	100	100	100	99	88	80	77	74	73	73	75	76	76
	75	100	100	100	100	100	98	86	79	74	71	70	70	72	72	72

Here we present the calculated expected % bias for each simulation we undertook in this study.

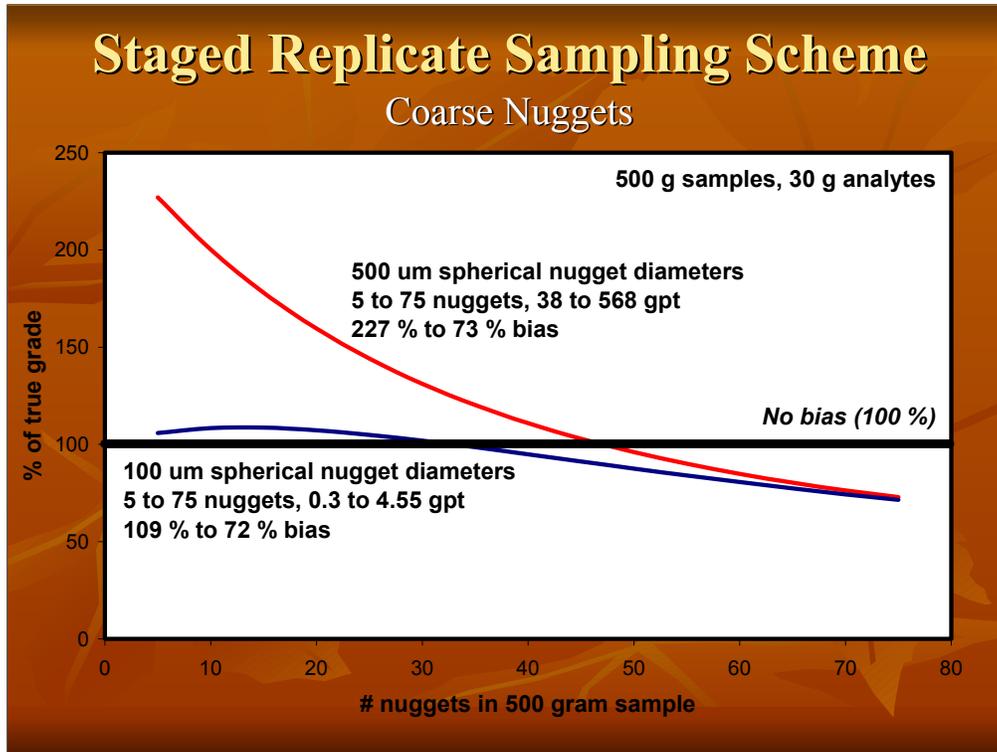
For small nugget sizes (in green), no significant bias is produced by the grade-based, staged sampling and analysis regimen.

However, as nugget sizes get larger, significant bias results.

When a large number of large nuggets per sub-sample exist, this staged sampling and analysis regimen results in significant under-estimation of the true grade.

Conversely, when a small number of large nuggets per sub-sample exist, this staged sampling and analysis regimen results in significant over-estimation of the true grade.

Note that the baseline case under-estimated the grade by 5 %, as depicted on the previous slide. Also note that the bias caused by this analysis regimen ranges from 72 to 188%. Is it no wonder why some gold mines over-produce their estimated grade, and some go out of business?



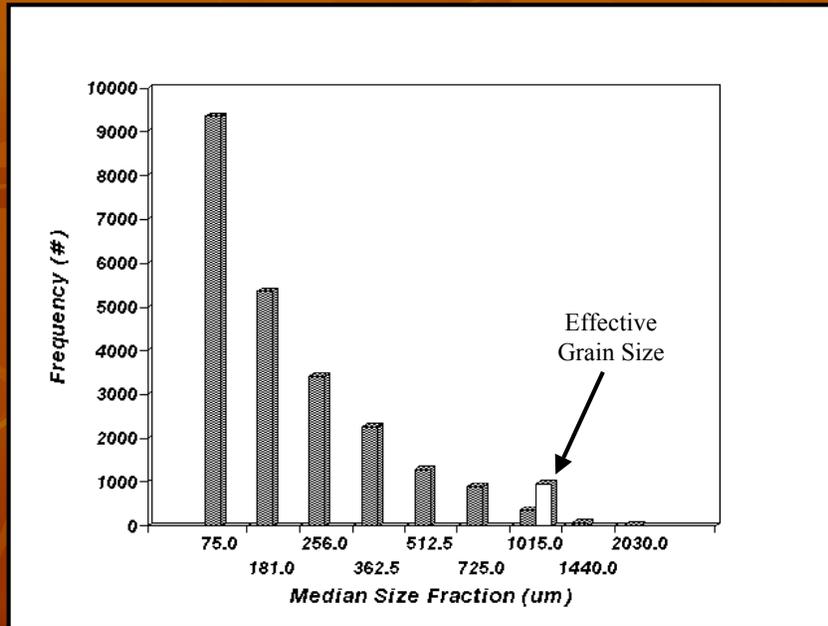
From these results, it is clear that grade estimation of coarse grained nugget-bearing mineral deposits will be the most affected by bias produced by a grade-based, staged sampling and analysis regimen.

To illustrate the magnitude of this bias for even larger nuggets, the expected biases for simulations with 100 um nuggets (in blue) and 500 um nuggets (in red) are presented on this graph.

Clearly, very significant biases can exist, and these can be in either direction (positive or negative), depending on the number of nuggets in the sample.

Without knowledge of the size or number of nuggets in a sample, one would never know whether a staged sampling and analysis regimen would be biased up or down. As a result, one cannot know whether the estimated grade of a deposit is the best or worst case scenario.

Coarse Nuggets



Unfortunately, real nuggets exhibit a range of grain sizes.

This frequency histogram (the dark grey bars) of diamond sizes from one zone at the Argyle Mine, Northern Territory, Australia depicts a typical range of nugget grain sizes in an ore.

If we sampled this material several times, the total number of carats would exhibit a variability due to the nugget effect. Some samples would contain more nuggets and some less. The largest contributor to this variability would be the largest nuggets, and how many of these occur in the samples will largely determine the grade of the samples.

We could obtain the same relative variability seen in the Argyle diamond ore containing a range of nugget sizes by sampling an idealized 'equant grain-bearing' material with only one nugget size. This 'effective' nugget size is represented by the white bar on this histogram.

Coarse Nuggets

- The ‘effective grain size’ of nuggets in samples are typically ~ 2/3 the size of the largest nugget (because the nugget sizes are mass weighted, the largest nugget exerts the largest ‘nugget effect’)

$$d = \sqrt[3]{\sum_{i=1}^n \frac{m_i}{m_t} d_i^3} \quad (\text{Clifton et al. 1969})$$

- Samples with coarse nuggets (typically those with the highest grades) exhibit the most bias (+ and -)
- As a result, the most grade bias is commonly in the most critical (high grade) samples!

The ‘Equant Grain Model’ and Poisson statistics allow one to calculate the ‘effective grain size’ of nuggets that via random sampling would produce the same relative error as a real sample with a range of nugget sizes.

Clifton et al. showed over 30 years ago that this ‘effective grain size’ can be determined using this formula. This formula involves calculation of a ‘mass weighted’ geometric mean, and requires knowledge of the size distribution of nuggets in a real sample.

Because the ‘effective grain size’ is $\frac{1}{2}$ to $\frac{3}{4}$ the size of the largest real nugget grain, the size of the largest grains will control the magnitude of the nugget effect. Their presence will thus create the most bias if a grade-based, staged sampling and analysis procedure is employed. This adds further insult to injury!

Conclusions

- Conditional re-sampling of assays subject to the 'nugget effect' based on initial grade will result in significant estimation bias
- Bias may be positive or negative, depending on the grades observed, the grade thresholds used to schedule re-assaying, and the nugget sizes
- **To avoid this bias:**
 - Re-assay all samples in balanced program, regardless of grade
 - Undertake an assay procedure amenable to large sample volumes (*e.g. – metallics; after pulverizing, assay all coarse material and assay sub-samples of fine material, then reconstitute grade*)

In summary, we have shown that conditional, staged sampling and analysis regimens that treat samples differently based on grade will produce significant analytical bias.

This bias can be positive or negative, depending on the number of nuggets in the sample, the grade thresholds used to schedule re-assaying, and the size of the nuggets.

In order to avoid this bias in samples influenced by the 'nugget effect', geologists, engineers, and metallurgists (note we have omitted geochemists from the above list, because they know better) should:

- 1) treat all samples identically by re-assaying all or a random selection of samples in a balanced program regardless of grade, and/or
- 2) use an analytical procedure that obtains its determination from a large sample volume (e.g. - metallics assay procedure), to avoid or mitigate the 'nugget effect'.

Thank you



Thank You.