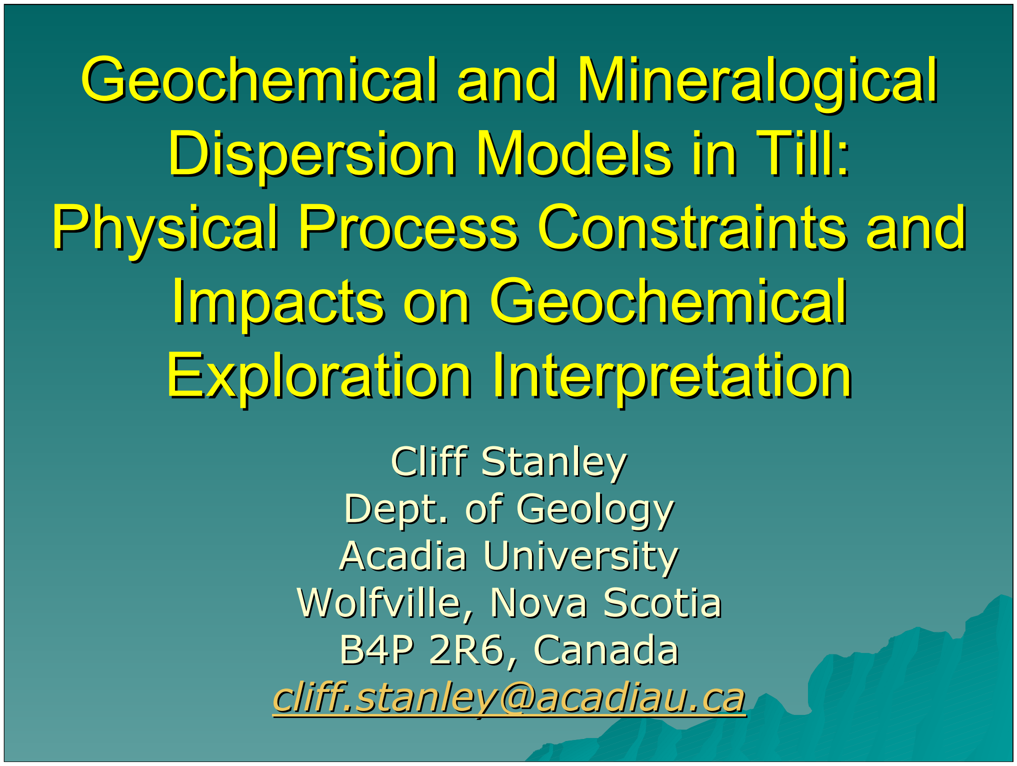
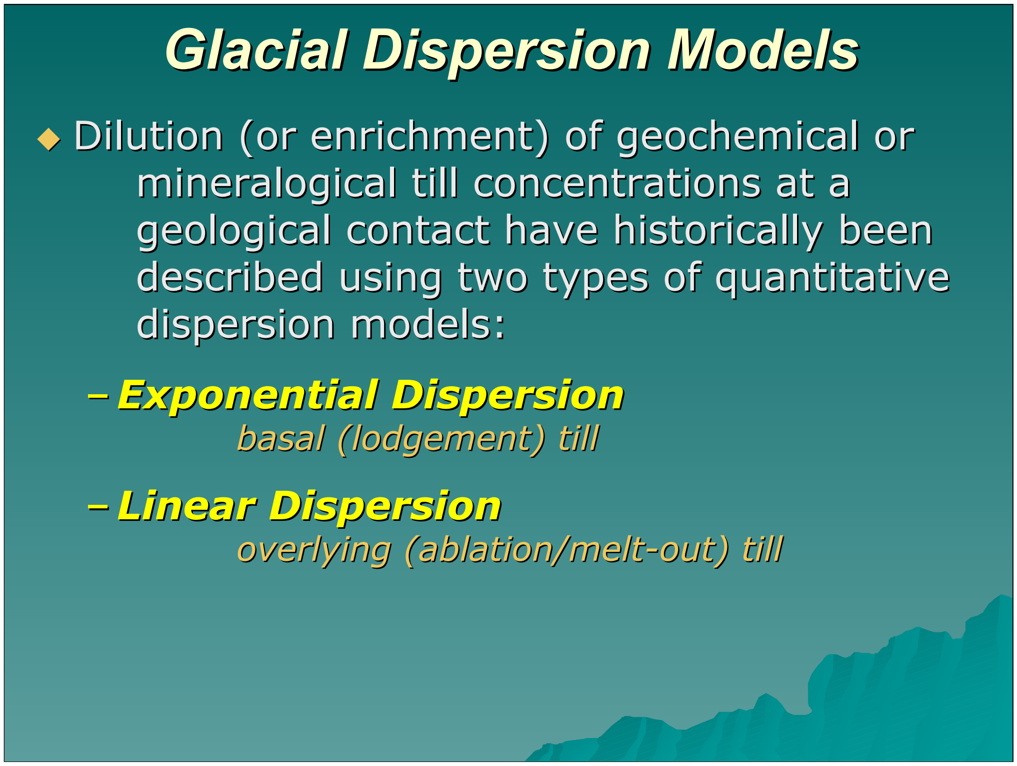


Thank you and Good Afternoon,

I have only recently begun to look the quantitative aspects of geochemical dispersion in till, largely through thesis research by my graduate student Andrea Locke. My limited experience in this subject requires that I examine previous glacial dispersion research results in substantial detail.

What I have discovered is that some of the commonly held beliefs regarding quantitative descriptions of glacial till dispersion may not be physically reasonable.

As a result, what I would like to do today is investigate the relationship between the mathematical models used to describe how geochemical concentrations in till change down-ice from a geological contact and the physical processes that create that till.

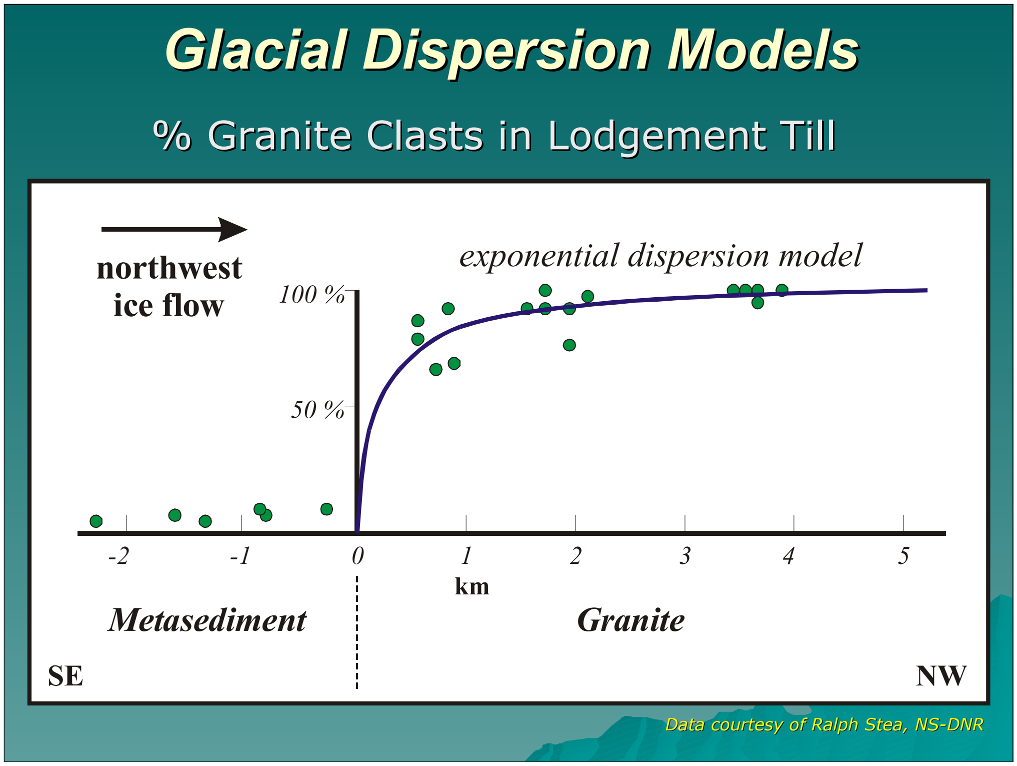


At a geological contact, till geochemical concentrations from the up- and down-ice lithologies will be diluted and enriched, respectively, by the introduction of material from the down-ice lithology.

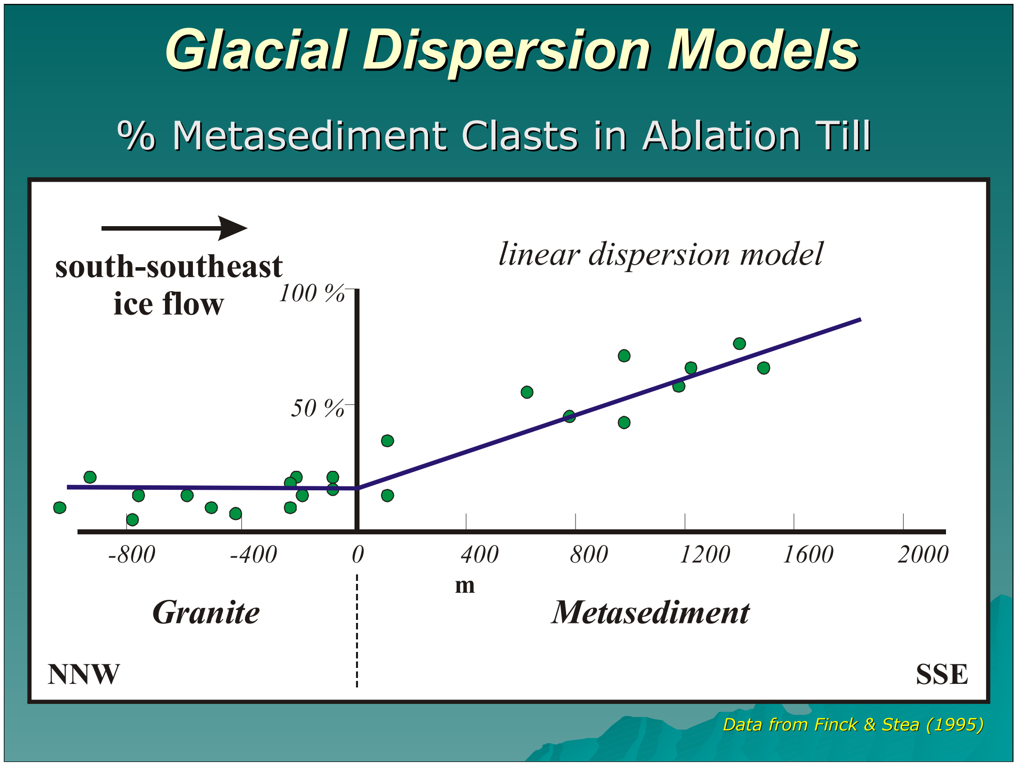
Two models have been used to describe this dilution and enrichment:

1) an exponential model, and
2) a linear model.

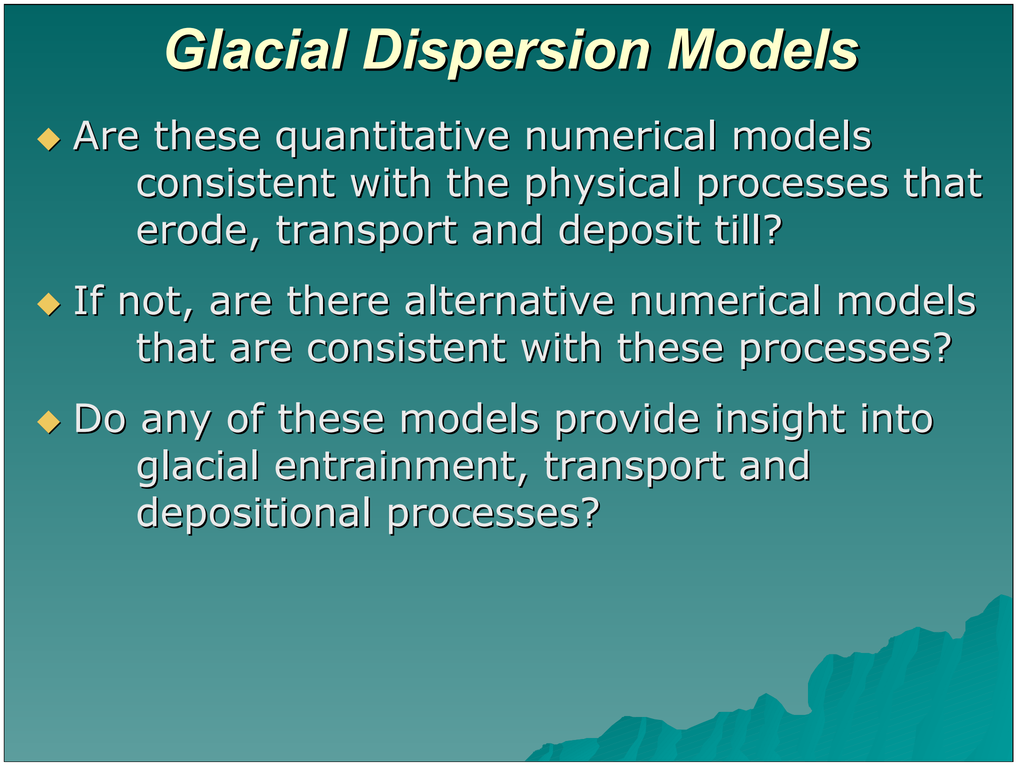
Both exponential and linear dispersion patterns have been observed in nature, and there is some suggestion that exponential dispersion patterns occur in lodgement till settings, whereas linear dispersion patterns have been observed in ablation till settings.



This example illustrates how the concentrations of granite clasts in the till increase across a meta-sedimentary rock-granite contact. These data have been regressed using an exponential dispersion model.



Similarly, this example illustrates how the concentrations of metasediment clasts in the till increase across a granite-meta-sedimentary rock contact. These data have been regressed using a linear dispersion model.

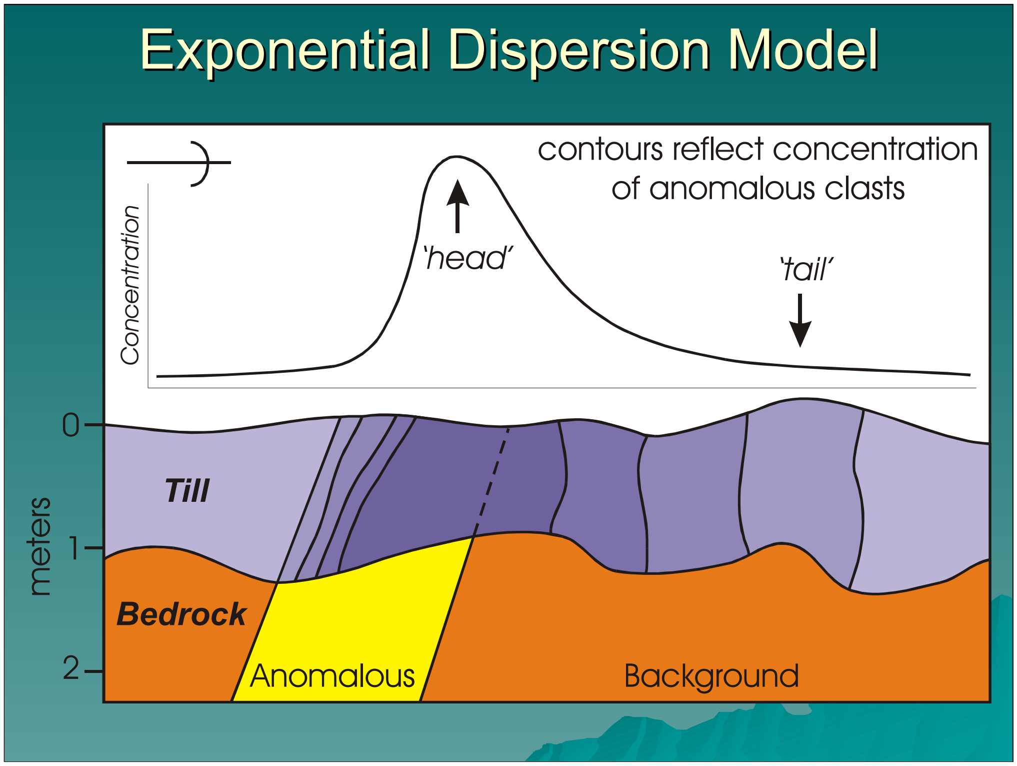


Although these exponential and linear models satisfactorily fit the data, three questions remain:

1) are these models consistent with the physical processes that produced the associated till dispersion patterns?

2) if these models are inconsistent with the physical processes, are there alternative models that are consistent and that would also satisfactorily fit the data?

3) does an evaluation of the relationship between these models and physical processes provide insight into glacial entrainment, transport and depositional processes?

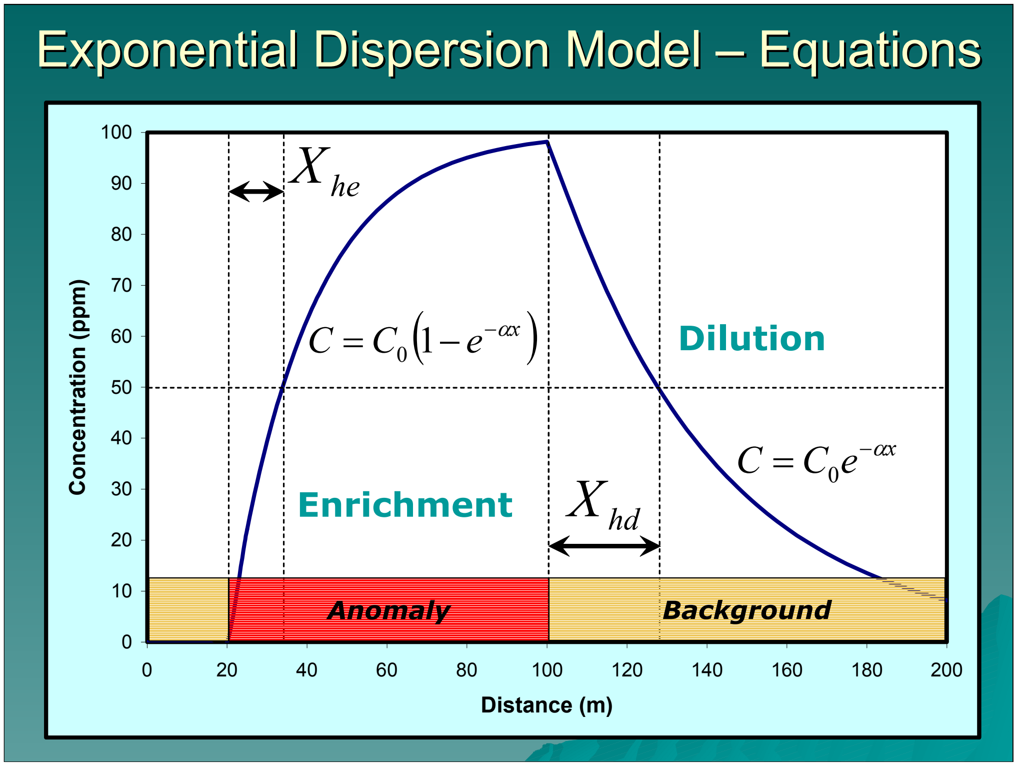


Let us first consider the exponential dispersion model.

This model is typically is invoked to explain lithological, mineralogical and geochemical anomalies, including their enrichment and dilution, over an exotic or anomalous lithology.

Concentrations rapidly increase over an anomalous lithology, then rapidly decrease down-ice from the exotic lithology, forming a 'head' to the anomaly.

The concentrations then slowly decrease further down-ice, forming an anomaly 'tail'.

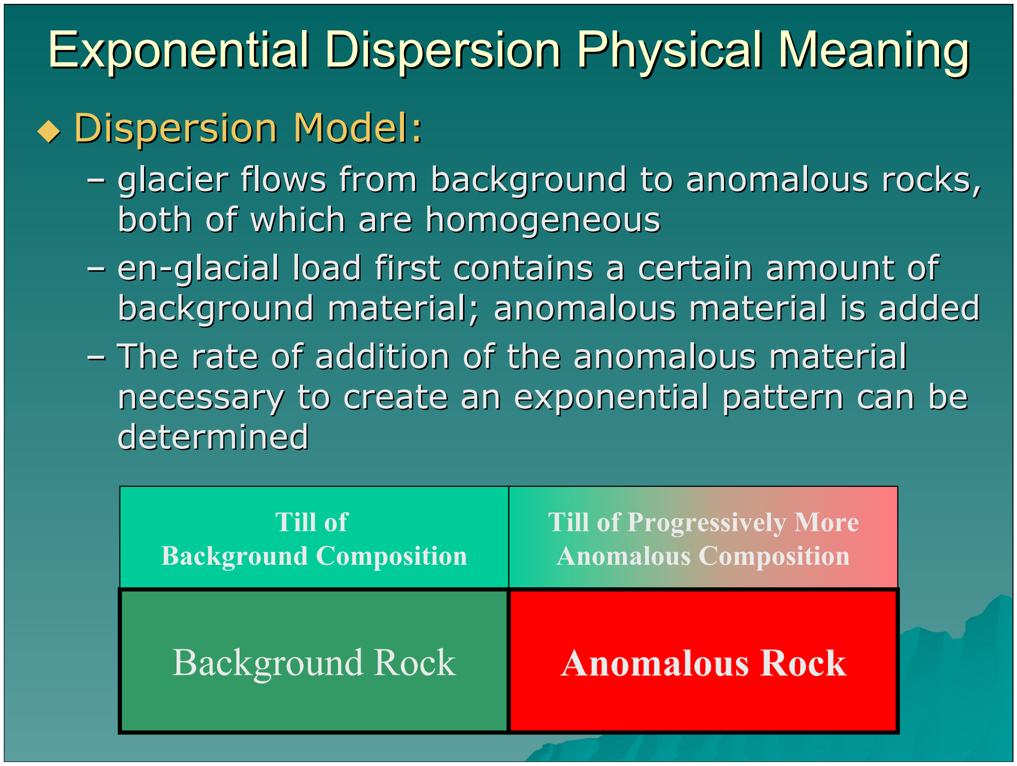


Numerical equations involving a negative exponential function have historically been used to describe the dilution and enrichment of the concentrations of anomalous and background material in the till.

These equations allow identification of ‘half distances’ for dilution and enrichment that are related to the ‘erodability’ of the bedrock. These ‘half-distances’ are critical in calculations of the location of the source of a till geochemical anomaly.

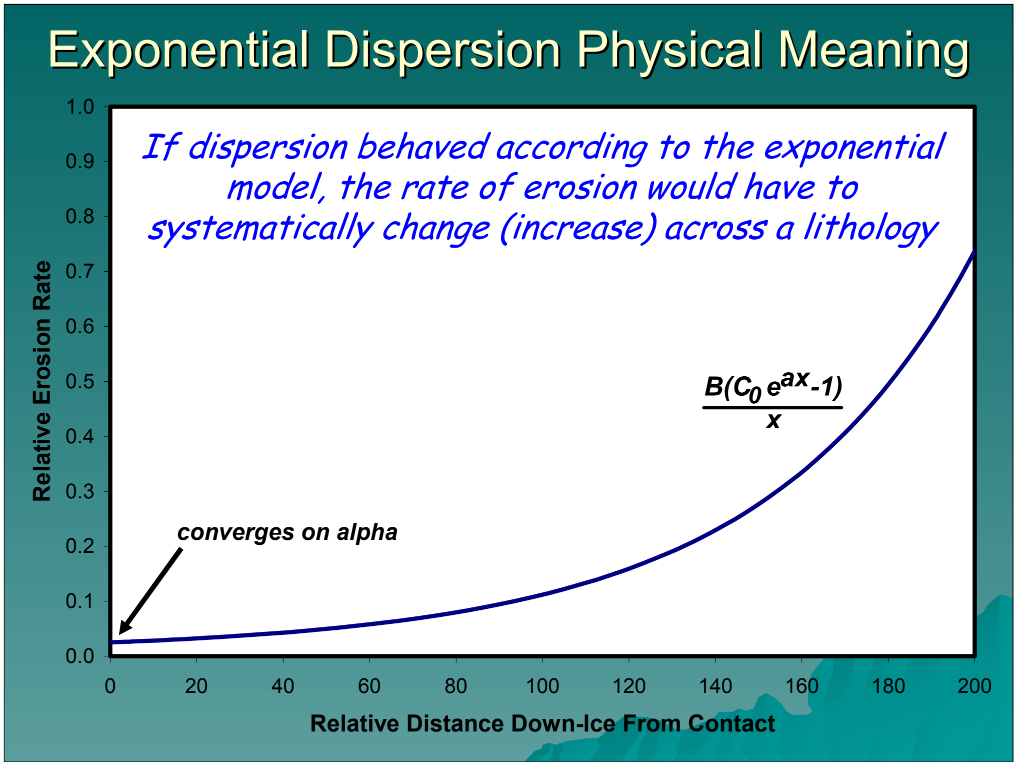
In this case, the enrichment half-distance is shorter than the dilution half-distance, indicating that the anomalous rock is more erodable than the background rock.

Unfortunately, my investigations of the negative exponential function indicate that it has specific numerical characteristics that probably make it inappropriate for use in describing till geochemical concentration patterns.



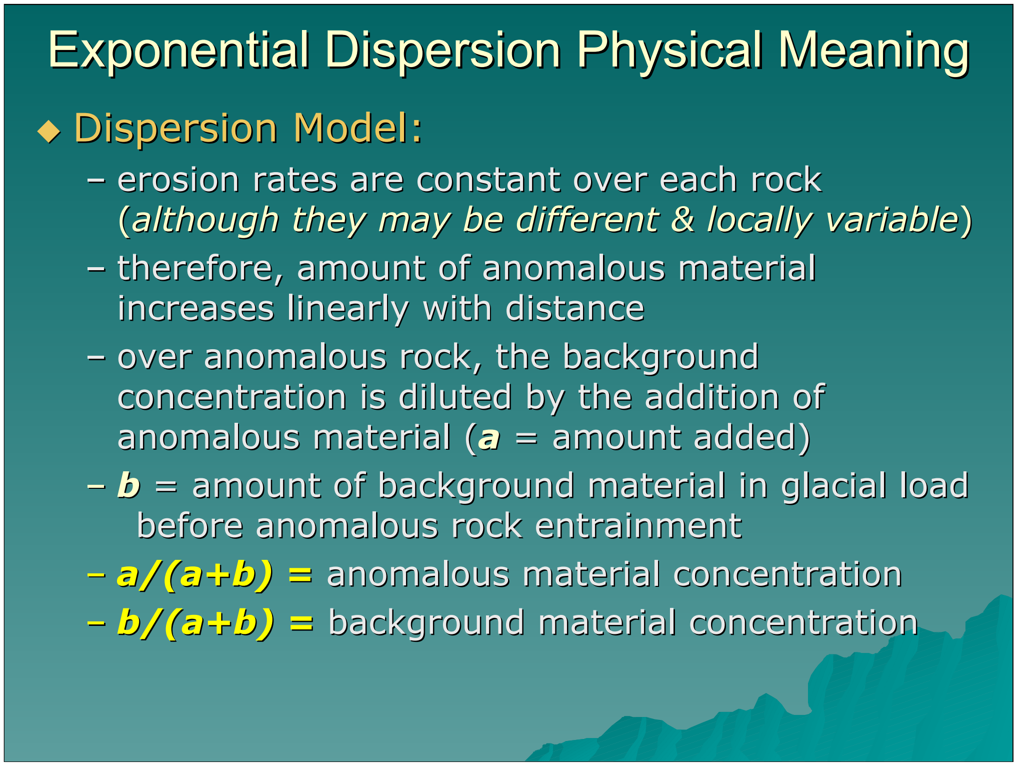
To illustrate my conclusions, consider a simple model where the initial en-glacial load should contains only background material. After flowing past the contact onto an anomalous rock, the anomalous lithology is eroded and added to the en-glacial load, eventually being re-deposited as till.

The rate of addition of the anomalous material necessary to create an exponential pattern can be determined using simple algebra.



The amount of material that must be added to the en-glacial load to create an exponential dispersion pattern is presented in this scale independent graph. Clearly, the erosion rate must increase significantly with distance (provided topographical variations are absent) to produce the exponential pattern.

This suggests that the exponential model is inconsistent with glacial erosion processes, as erosion rates can probably be expected to be relatively constant over a homogeneous lithology (at least on average).



What then is a realistic functional form for glacial dispersion enrichment and dilution?

In our simple model, erosion rates are constant over each rock, although they may be different & locally variable.

Thus, as the amount of erosion is approximately constant, the amount of anomalous material in the en-glacial load should increase linearly with distance from the contact.

This enriches the anomalous material concentration in the en-glacial load, and dilutes the background material concentration in the en-glacial load. These concentration changes are eventually manifested in the resulting till after deposition.

Concentrations of anomalous and background material can be determined by these formulae.

Exponential Dispersion Physical Meaning

Rock	0	1	2	3	4	5	*meters*
A	0	1	2	3	4	5	*amounts*
B	5	5	5	5	5	5	*amounts*
Total	5	6	7	8	9	10	
% A	*0*	*17*	*28*	*37*	*45*	*50*	***a/(a+b)***
% B	*100*	*83*	*72*	*63*	*55*	*50*	***b/(a+b)***

- Thus, this simple physical model defines an ***Inverse Dispersion Model:***

  a/(a+b) & b/(a+b)

If material is added to the en-glacial load ‘linearly’, an alternative model to exponential dispersion can be invoked to explain geochemical dilution and enrichment.

This model involves ‘inverse’ dispersion, because the ‘a’ in these equations (the amount of anomalous material added) is the only variable.

Inverse Dispersion Model

- ***This model is different from an exponential model!***

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} \cdots \neq \frac{c}{c+x}$$

- ***The Exponential Function:***
  - initially decreases slower than the inverse function
  - converges to 0 faster than the inverse function

The inverse function is generally similar to the exponential function in that it slowly approaches zero. Nevertheless, it is substantially different.

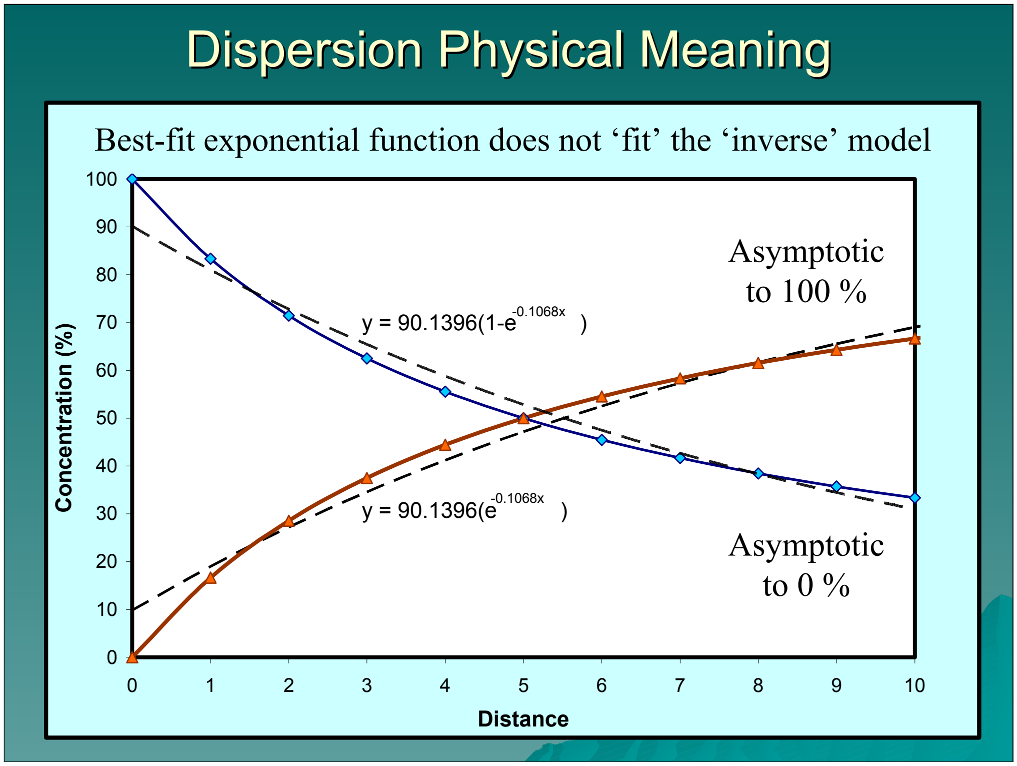
This can be demonstrated both mathematically and graphically.

Mathematically, there is no simple relationship between the exponential and inverse functions. Here I show that a serial expansion of the exponential function neither equals nor approximates the inverse function.

Graphically, the exponential function:

1) decreases slower initially than the inverse function, but

2) converges to 0 faster.



This is a graphical comparison of the inverse function and the 'best fit' exponential function. The differences between these functions are clear.

Note that because the exponential function converges to zero faster, the amount of erosion necessary to create exponential enrichment and dilution must increase down-ice, as demonstrated.

One of the principle calculations made using numerical dilution and enrichment models involves estimating the transport distance of exotic material observed in tills (i.e. – determining the anomaly source location).

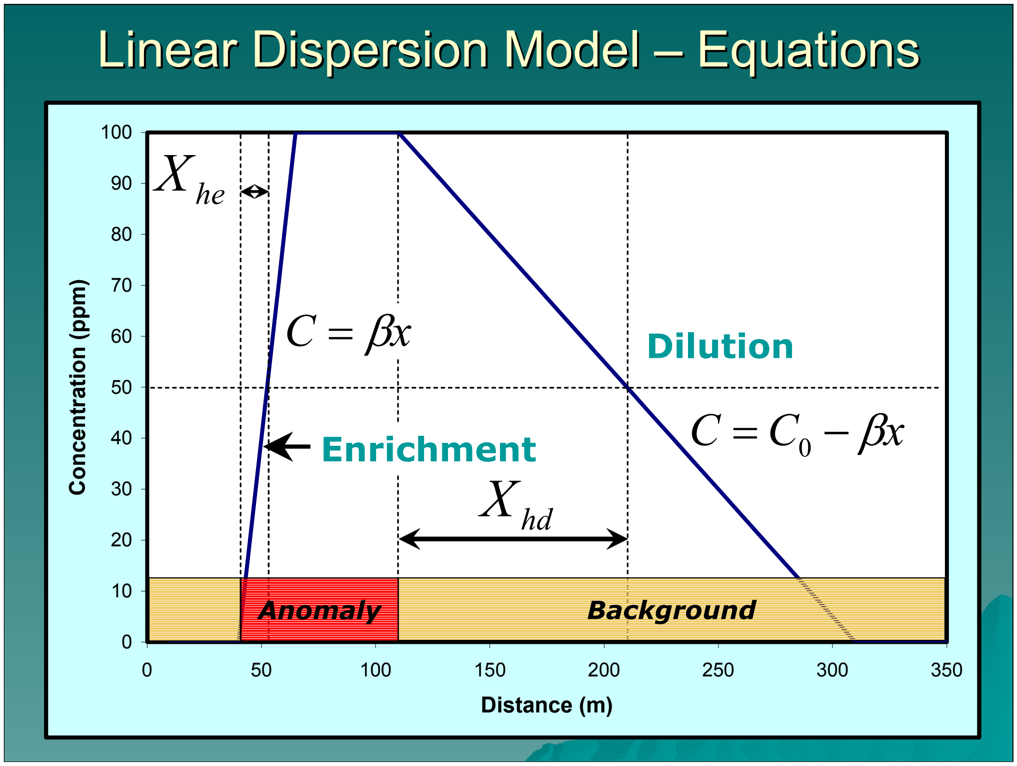
Because the exponential model is inconsistent with sub-glacial physical processes, its use in this application will provide inaccurate estimates of transport distance.

Obviously, this can have costly consequences in mineral exploration efforts.

Linear Dispersion Model

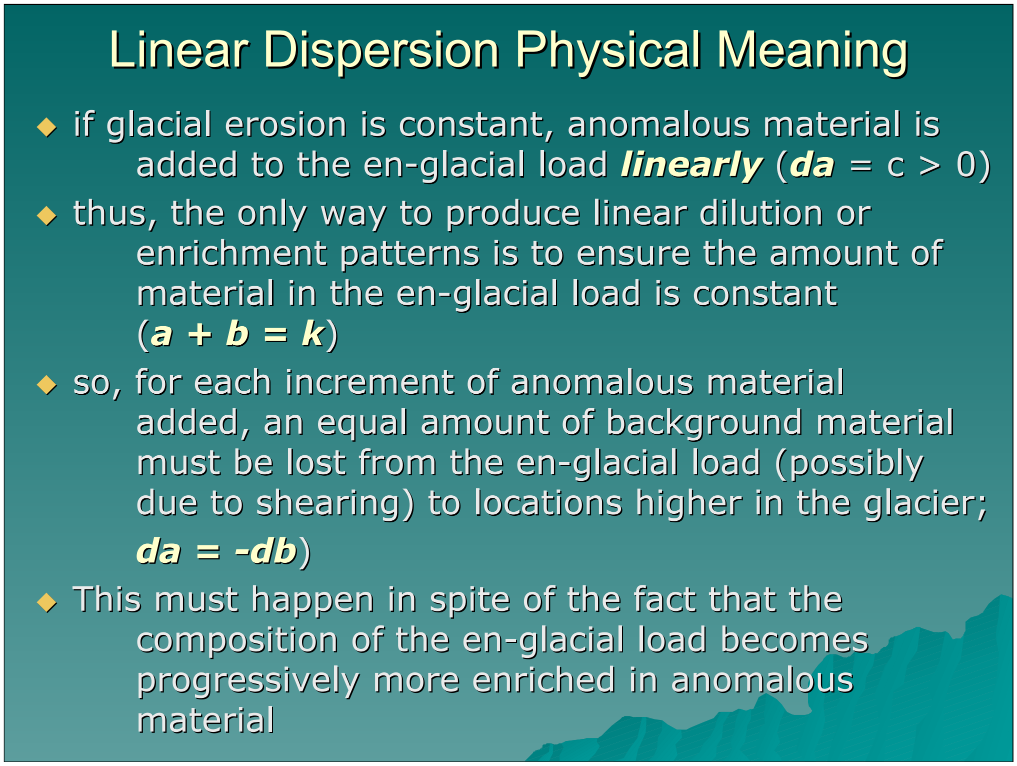
- What about the linear dispersion model?

Now, if we need to substitute an ‘inverse’ function for the ‘exponential’ function to create a dispersion model for lodgement tills that is consistent with physical process, what about the ‘linear’ dispersion model in ablation tills?



Like the exponential model, the linear dispersion model also has corresponding equations describing how concentrations increase and decrease down-ice from a geological contact.

As a result, we must ask how we could create a linear dilution or enrichment pattern in geochemical concentrations in till?



Recall that if glacial erosion is relatively constant, anomalous material will be entrained linearly into the en-glacial load.

Thus, the only way to obtain a linear pattern is to have the amount of material in the en-glacial load remain the same. This means that as the anomalous material is added to the en-glacial load through erosion, an equal amount of background material must be lost from the en-glacial load. You effectively add from the bottom and remove from the top!

Although this might seem reasonable at first, as the new material contributed through erosion could effectively force the already entrained material to locations higher in the glacier, this process must be highly selective, removing only the background material from the lower reaches of the glacier. Unfortunately, with distance from the geological contact, the composition of the en-glacial material becomes more and more concentrated in anomalous material, making removal of only the background material more and more difficult.

Linear Dispersion Physical Meaning

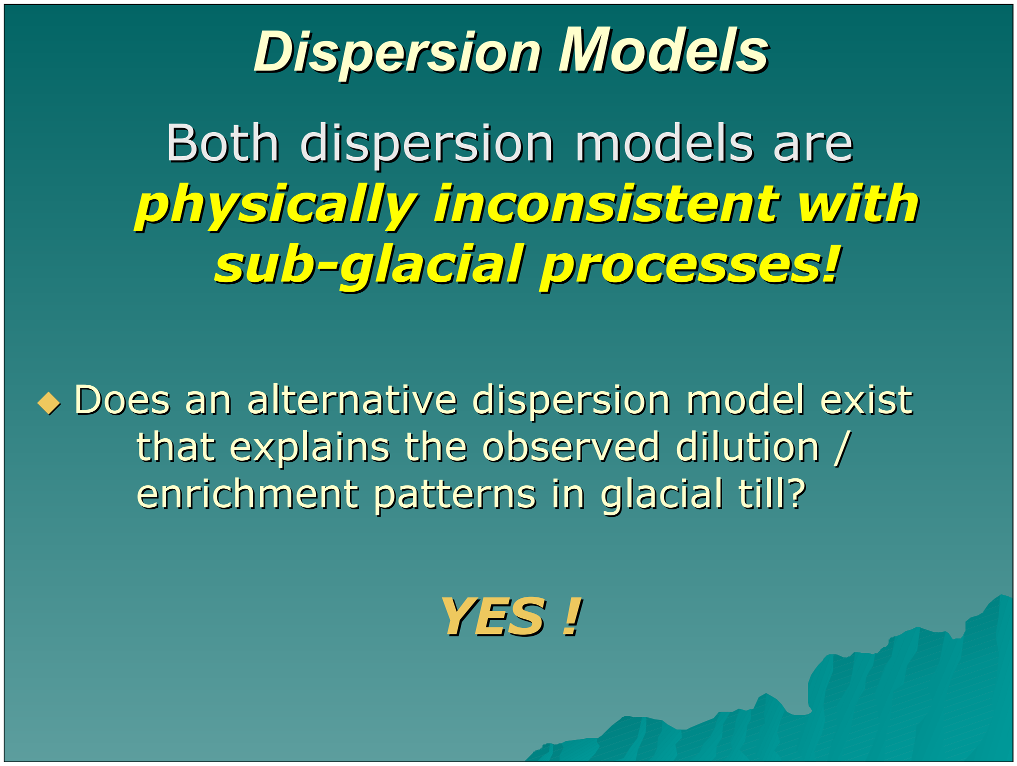
Rock	0	1	2	3	4	5	*meters*
A	0	1	2	3	4	5	*amounts*
B	5	4	3	2	1	0	*amounts*
Total	5	5	5	5	5	5	
% A	*0*	*20*	*40*	*60*	*80*	*100*	***a/k***
% B	*100*	*80*	*60*	*40*	*20*	*0*	***b/k***

- ***a/k*** & ***b/k*** are linear decay sequences
- To create a linear decay, only background material can be removed from the en-glacial load; unfortunately, this load becomes progressively more enriched in anomalous material

Improbable!

As a result, the linear model describing enrichment and dilution across a geological contact is also inconsistent with the physical processes responsible for till formation.

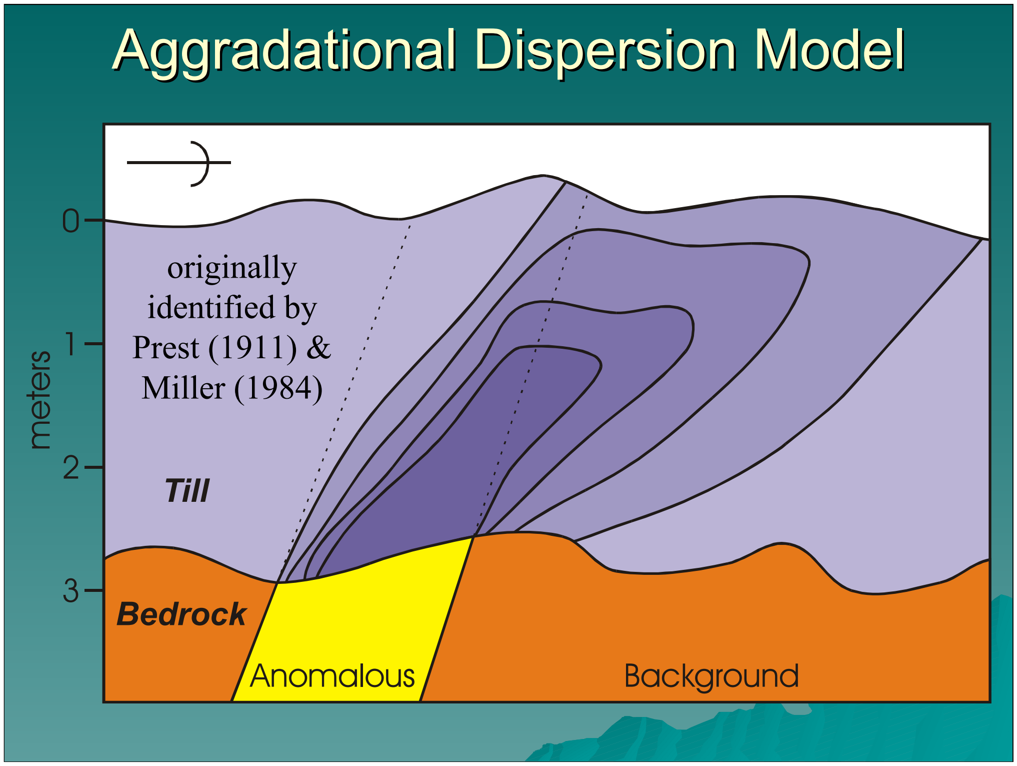
The required selectivity is highly improbable!



Consequently, both numerical models that have been used historically to describe concentration changes down-ice from a geological contact are inconsistent with the physical processes responsible for till formation.

Is there a numerical model that can be used in lieu of these models?

YES!



The model I have developed to replace the exponential and linear dispersion models is referred to as the ‘aggradational dispersion model’. It is designed to explain both ‘exponential’ and ‘linear’ dispersion patterns simultaneously, and involves a multi-layer formulation to describe the concentrations of rocks, minerals or elements in till.

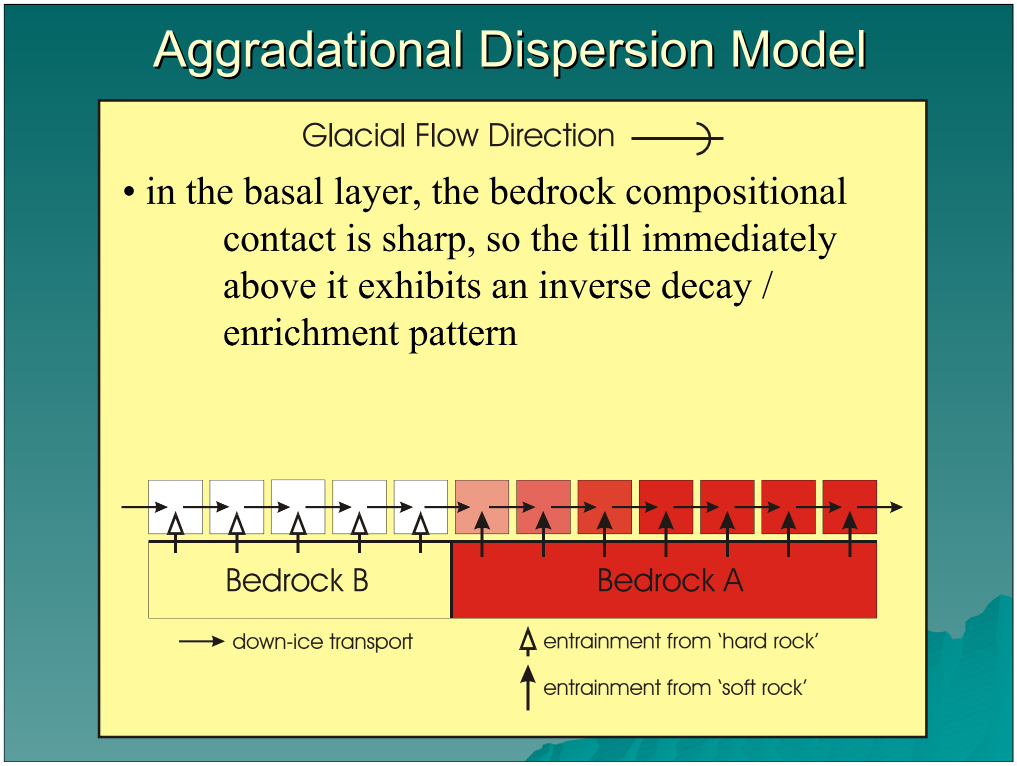
As you will see, this model is consistent with the cross-sectional patterns observed in tills (including those originally recognized by Prest in 1911, and more recently by Miller in 1984, and by many others over the last 15 years).

These patterns involve a dispersion train that exhibits:

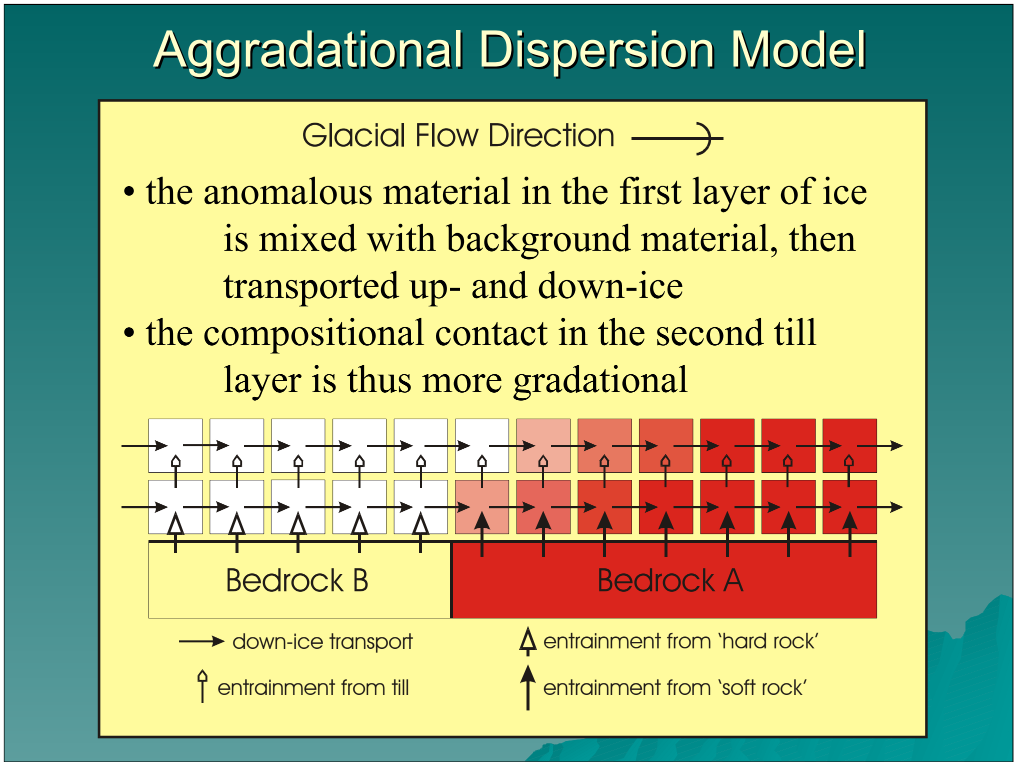
1) a shallow ascent to the till surface,

2) increasingly diffuse boundaries, and

3) dilution through the core of the dispersion train.

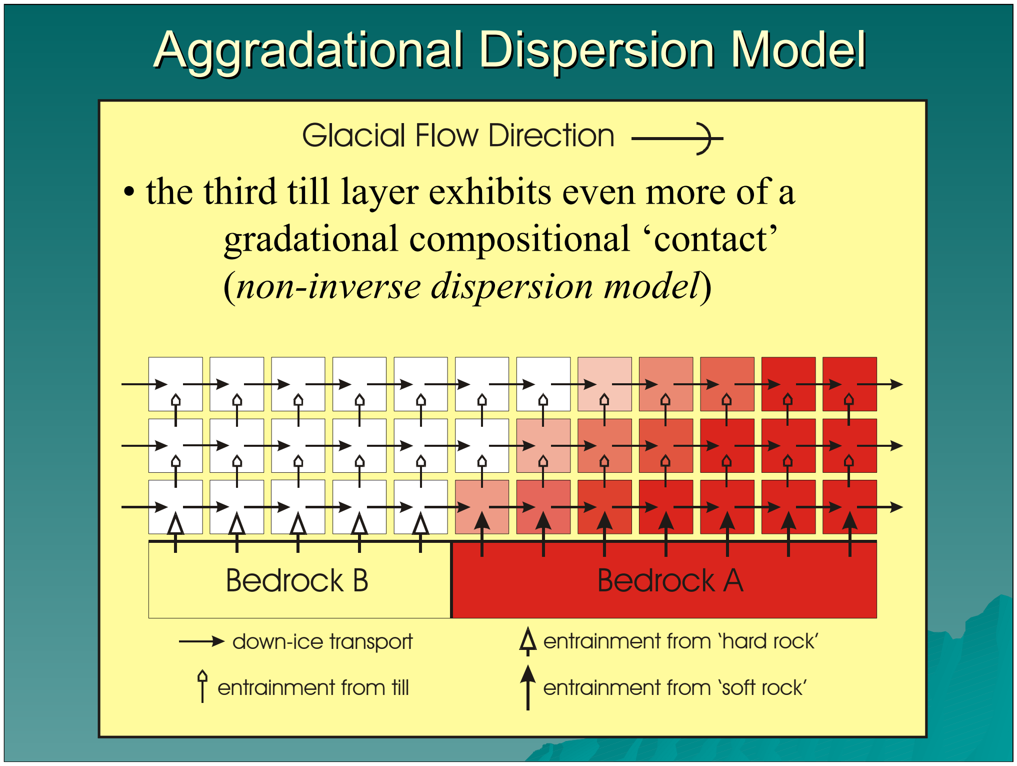


The model starts with erosion of bedrock in a linear manner (as described previously), resulting in a basal layer of till that exhibits an 'inverse' dilution and enrichment pattern.

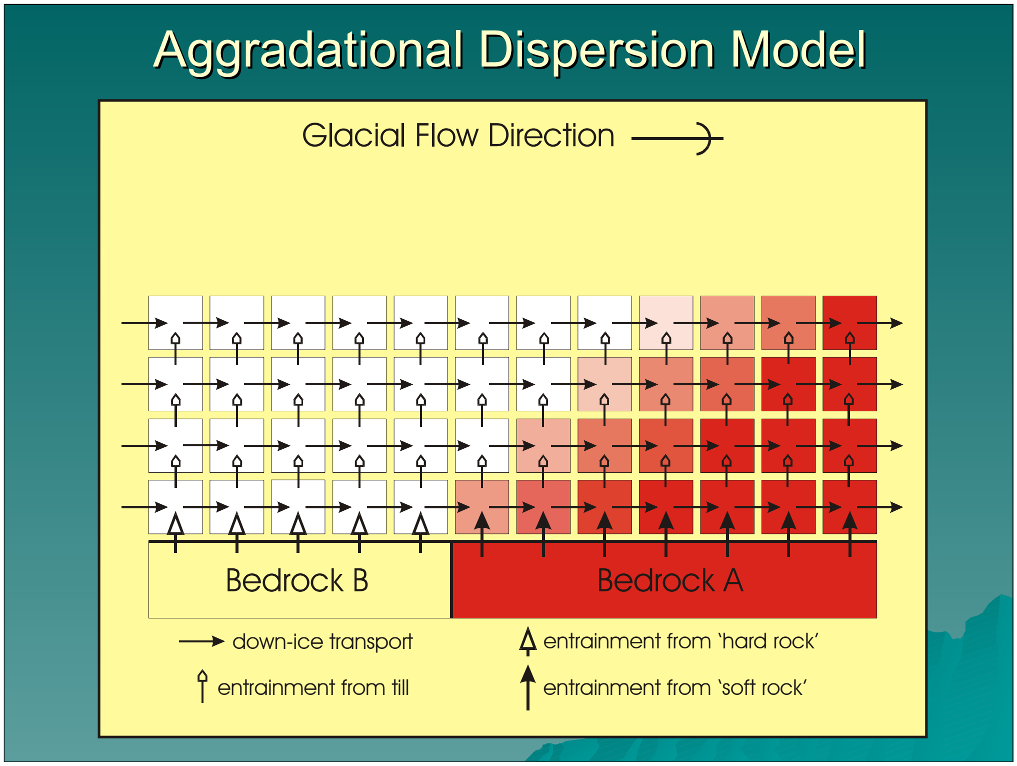


The material entrained into the basal layer of ice from below mixes (to some extent) with material transported down-ice, and subsequently effectively 'forces' previously entrained material higher into the ice (via regelation-induced accretion at the bedrock-ice contact).

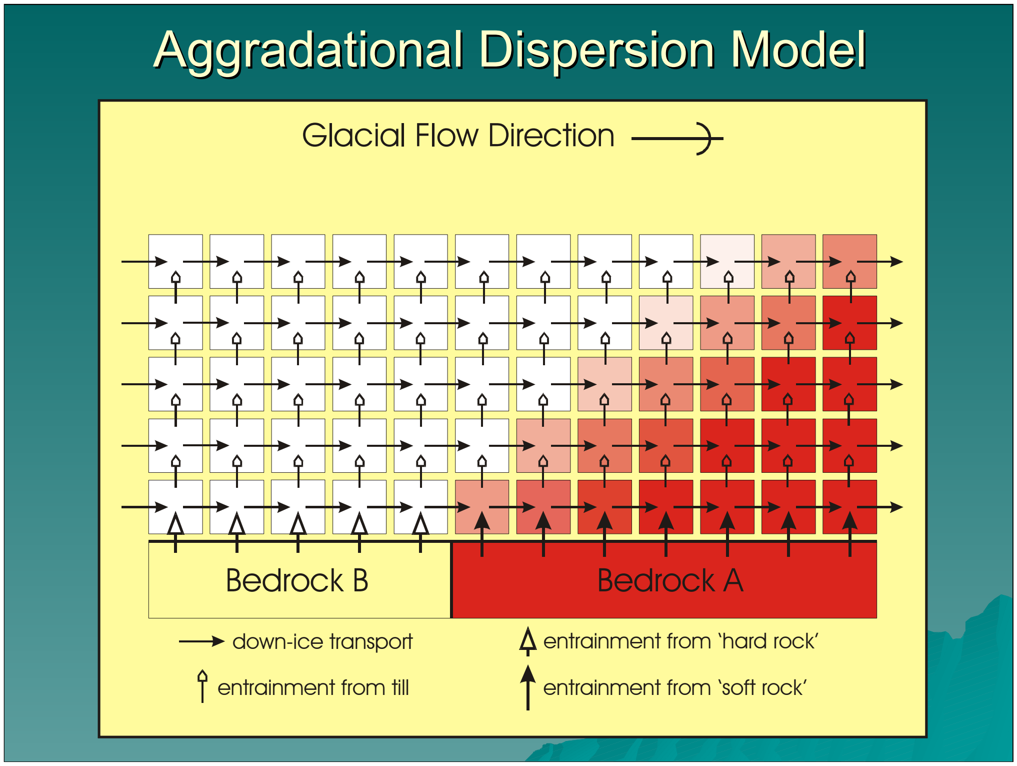
This material enters a second overlying layer and, because of prior mixing, exhibits a more gradational compositional 'contact'.



This process is repeated to form a third layer, which exhibits an even more gradational compositional 'contact'.


Aggradational Dispersion Model
Glacial Flow Direction
Bedrock B
Bedrock A
down-ice transport
entrainment from 'hard rock'
entrainment from till
entrainment from 'soft rock'

Et cetera.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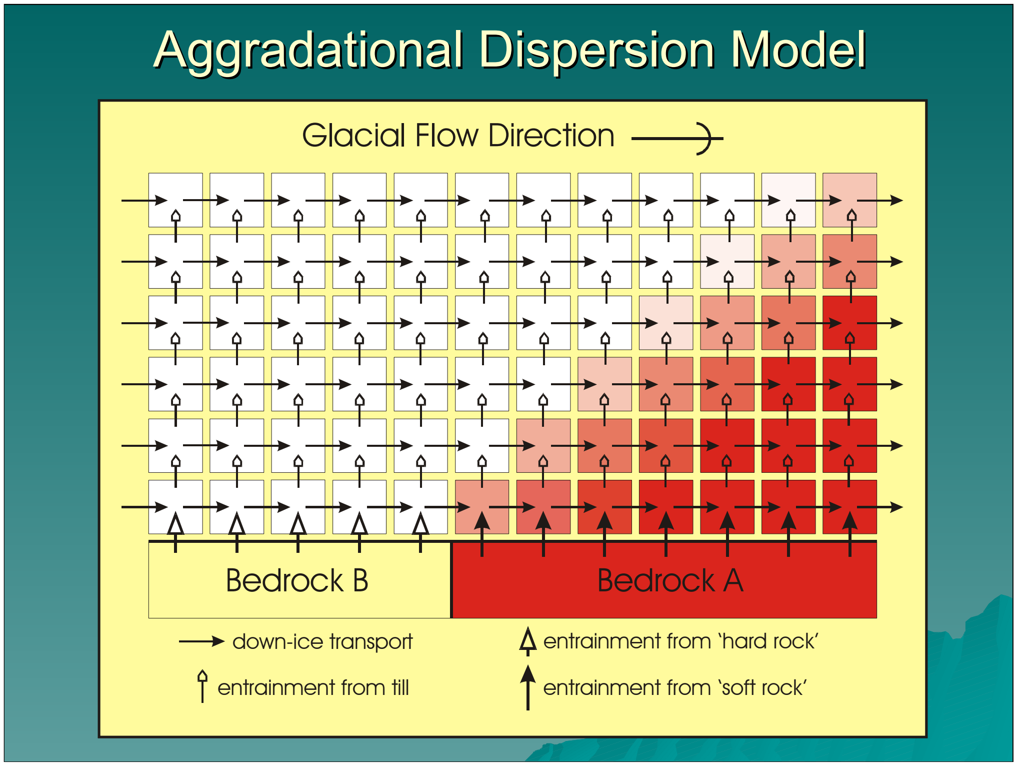
Et cetera.



Et cetera.

Ultimately, this results in a multi-layer till dispersion model.

Aggradational Dispersion Model

Distance		0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300
Till Layers	10	0	0	0	0	0	0	0	0	2	3	6	8	10	11	11	10
	9	0	0	0	0	0	0	0	1	3	5	8	10	11	11	11	9
	8	0	0	0	0	0	0	0	2	5	8	10	12	12	11	10	8
	7	0	0	0	0	0	0	1	4	8	11	13	13	12	10	8	6
	6	0	0	0	0	0	0	3	7	11	14	14	13	11	8	6	4
	5	0	0	0	0	0	1	6	12	16	16	14	11	8	6	4	3
	4	0	0	0	1	1	2	12	18	18	16	12	8	6	4	2	1
	3	0	0	1	3	6	9	21	22	18	13	8	5	3	2	1	0
	2	0	2	7	13	19	26	30	21	13	7	4	2	1	1	0	0
	1	0	17	32	44	53	61	27	12	5	2	1	0	0	0	0	0
Bedrock		**0**	**100**	**100**	**100**	**100**	**100**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Background | *Anomaly* | *Background*

- anomalous material is smeared down-ice
- width of the anomaly increases down-ice
- concentrations of most anomalous material decrease down-ice (diluted by mixing)
- concentration patterns differ in each layer of the till

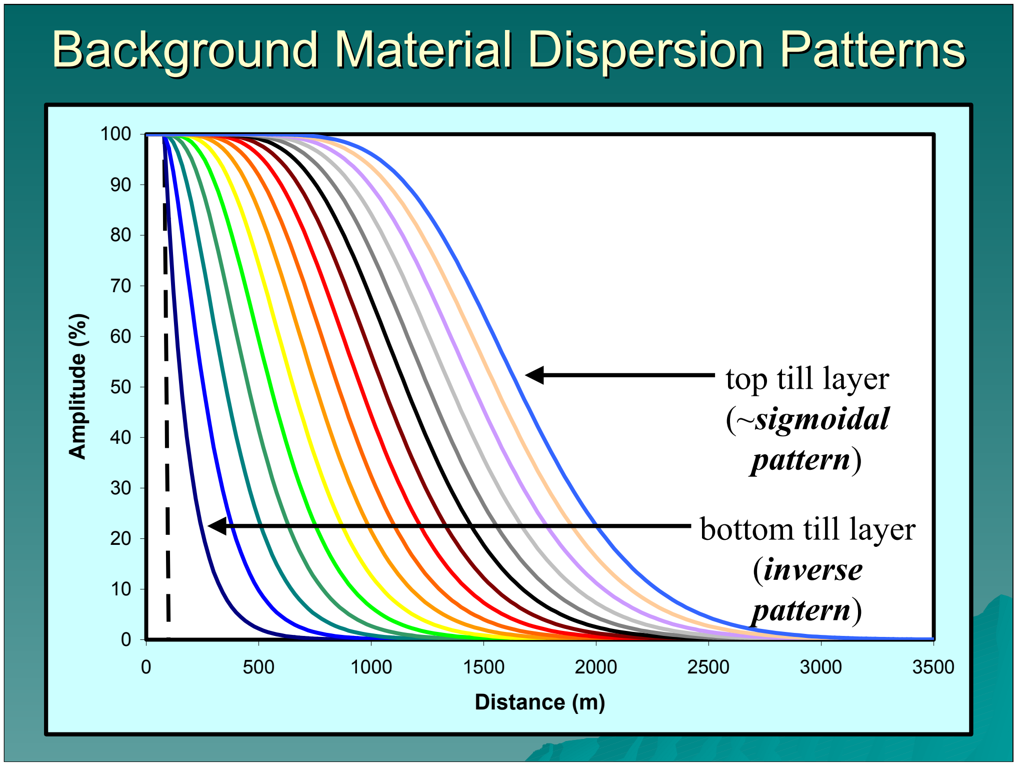
In this finite difference numerical simulation of the Aggradational Dispersion Model, the dispersion train created by this process has the following characteristics:

1) a shallow ascent through the till to the surface,

2) exhibits increasingly diffuse boundaries, and

3) exhibits dilution through the core of the dispersion train.

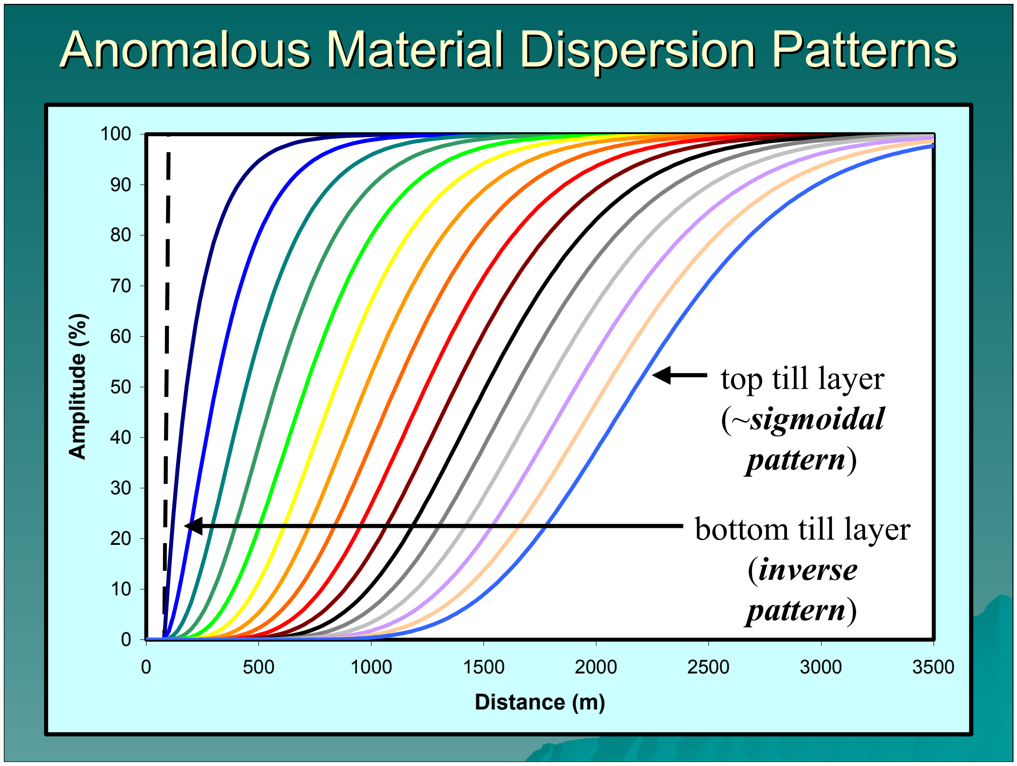
Thus, it produces cross-sectional patterns that are identical to those observations of Prest (1911) and Miller (1984).



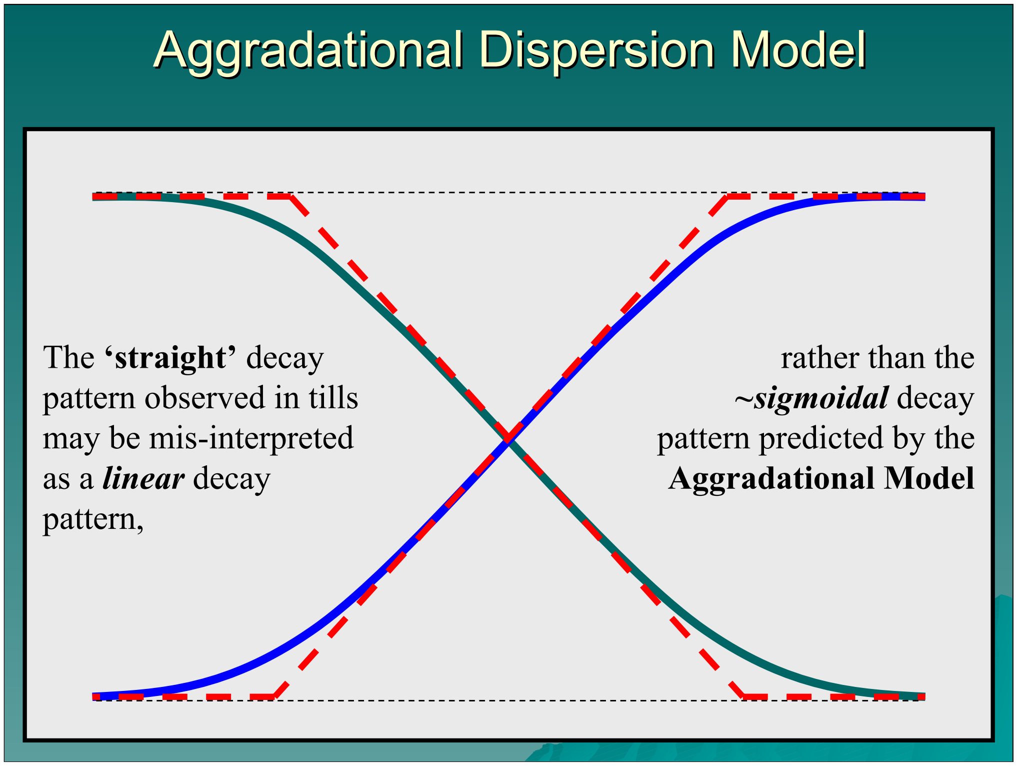
The background compositional changes in each layer of till in this model (undergoing dilution) are substantially different.

The basal till exhibits variation identical to that of an inverse model.

Higher in the till, the compositional contact becomes more and more gradational, eventually exhibiting an approximately sigmoidal form.



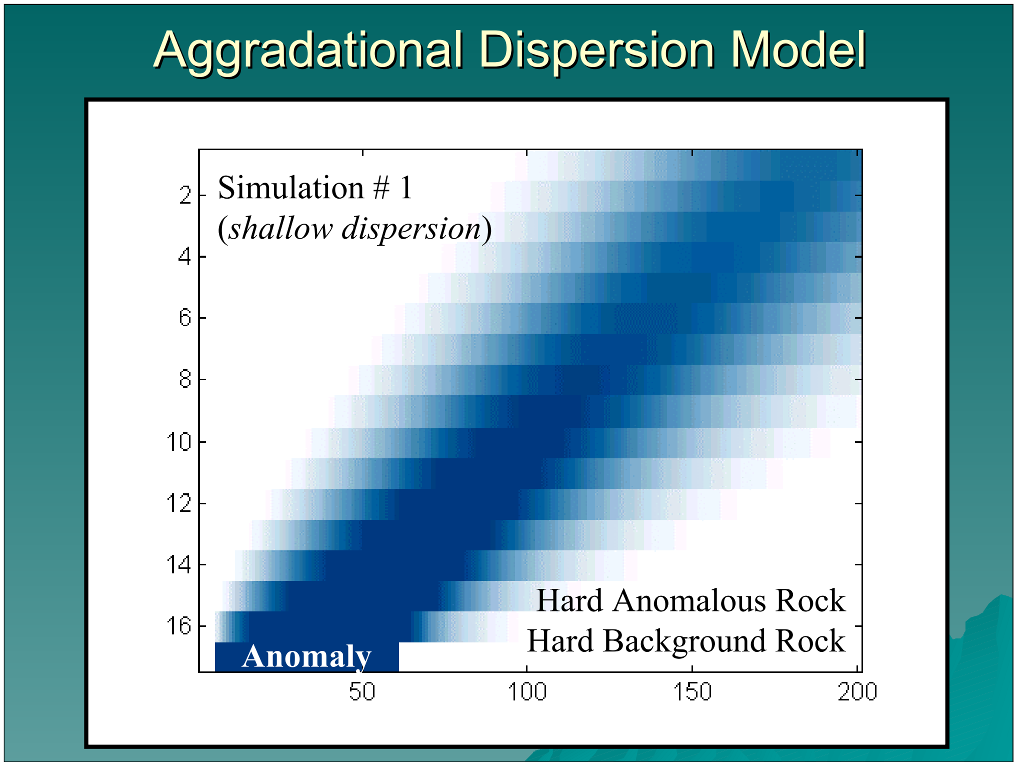
Analogous compositional changes (involving enrichment) occur for the anomalous lithology.



In the same way that inverse dispersion patterns have been mistaken for exponential dispersion patterns, the sigmoidal dispersion pattern predicted by the model could easily be mistaken for a linear dispersion pattern.

As a result, the Aggradational Dispersion Model accommodates both observed till dispersion patterns, and the model predicts these to occur at different but apropriate levels in the till.

Inverse (pseudo-exponential) dispersion patterns occur in the basal parts of tills (i.e. - lodgement till settings), whereas sigmoidal (pseudo-linear) dispersion patterns occur in the upper parts of till (i.e. - ablation till settings). This is consistent with the locations where the various empirical dispersion patterns are observed in nature.

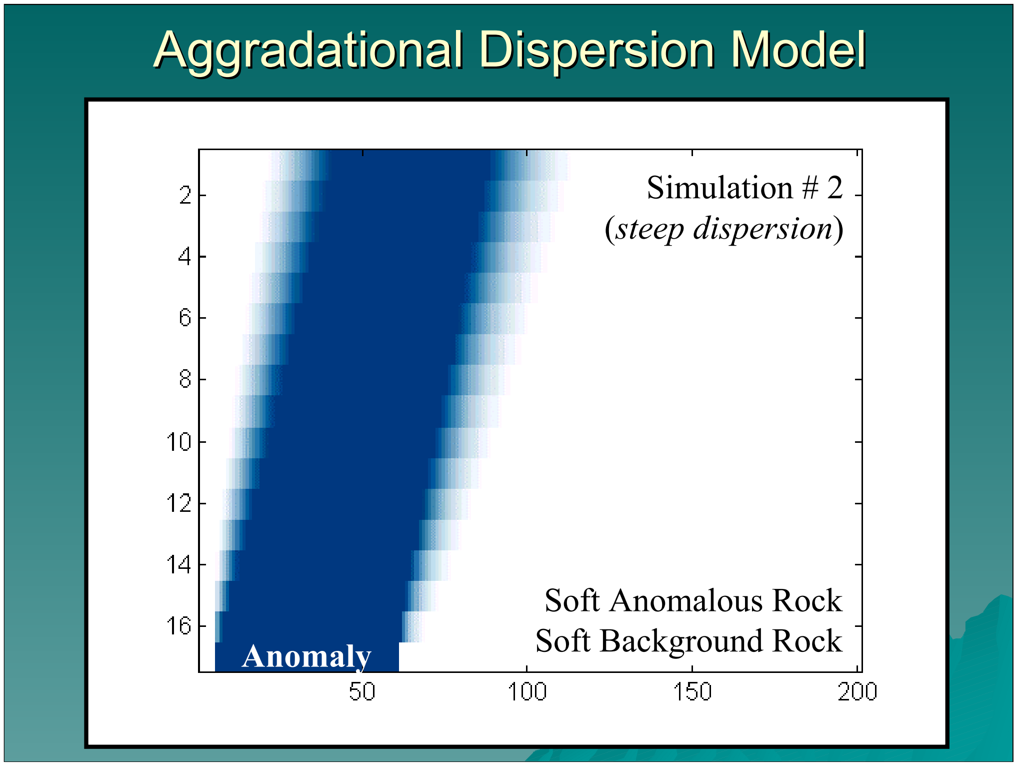


Any model that has quantitative predictability, and thus validity, should identify consequent characteristics that form the bases of tests of the model.

By changing the various input parameters of the Aggradational Dispersion Model, one can determine what variations one might see in till cross-sections. These are some of these consequent characteristics.

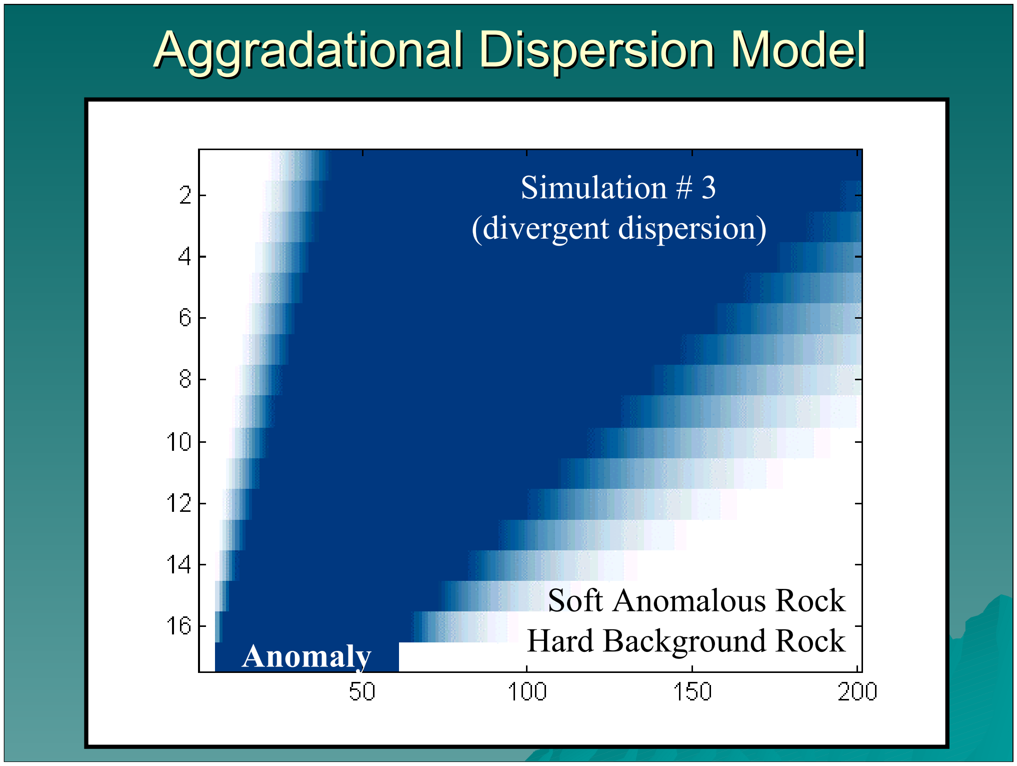
In this simulation, relatively hard background and anomalous rocks were over-ridden by a glacier, eroded, transported and deposited as till.

The resulting dispersion train climbs relatively slowly down-ice. In doing so, it becomes more diffuse down-ice, and also more diluted.



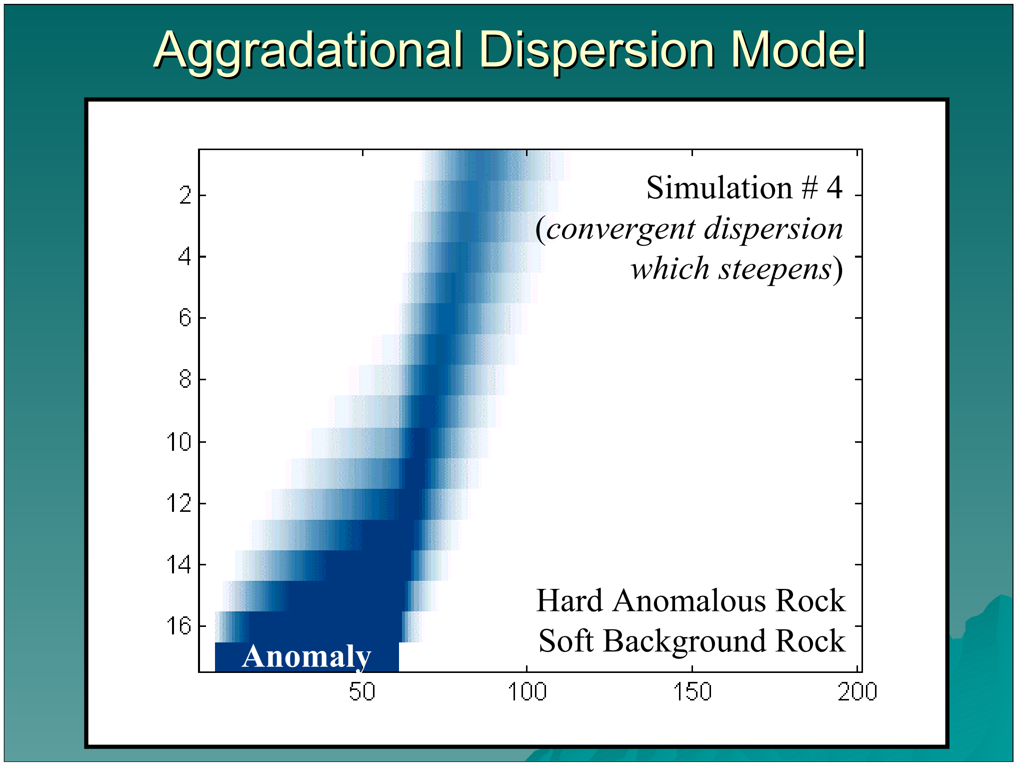
In this simulation, softer, more erodable background and anomalous rocks were considered.

In this case, the dispersion train climbs more rapidly through the till, the boundaries of the dispersion train become less diffuse, and the concentrations in the dispersion train do not dilute as much.



This simulation was made with relatively soft, erodable anomalous rock, and a hard background rock.

In this case, the dispersion train diverges down-ice, becomes more diffuse on its down-ice edge but is less diffuse on its up-ice edge, and does not exhibit significant dilution.

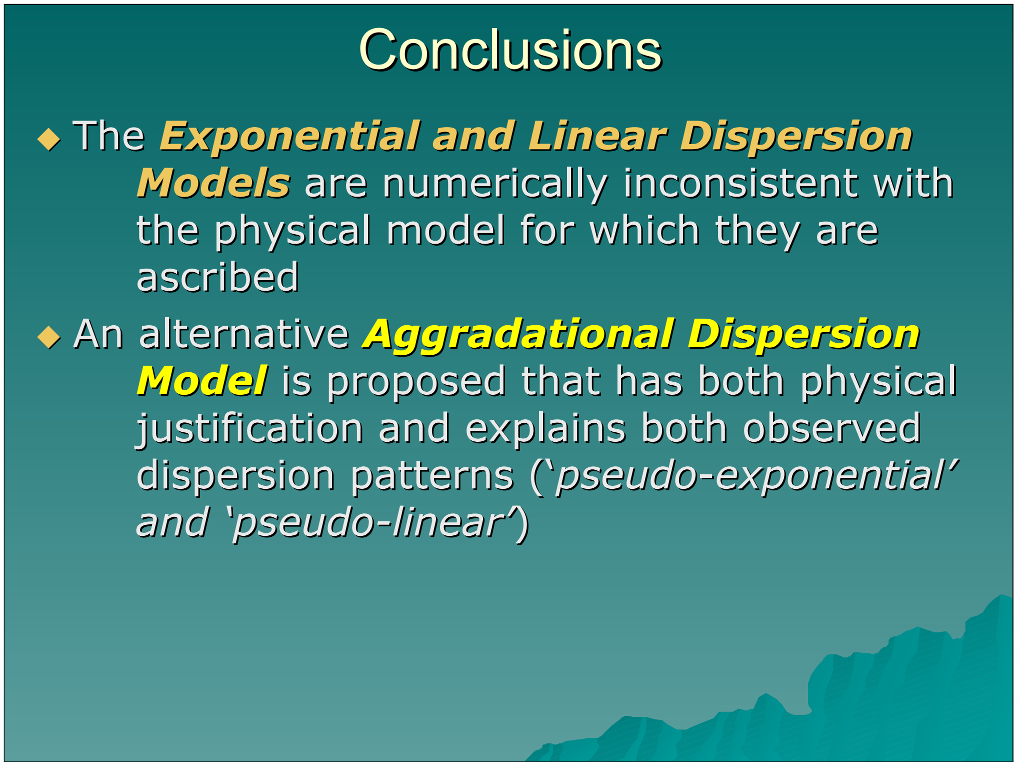


Finally, this simulation was made with a relatively hard anomalous rock, and a soft, erodable background rock.

In this case, the dispersion train converges down-ice, is more diffuse on its up-ice edge and less diffuse on its down-ice edge, exhibits significant dilution, and bends upward through the till.

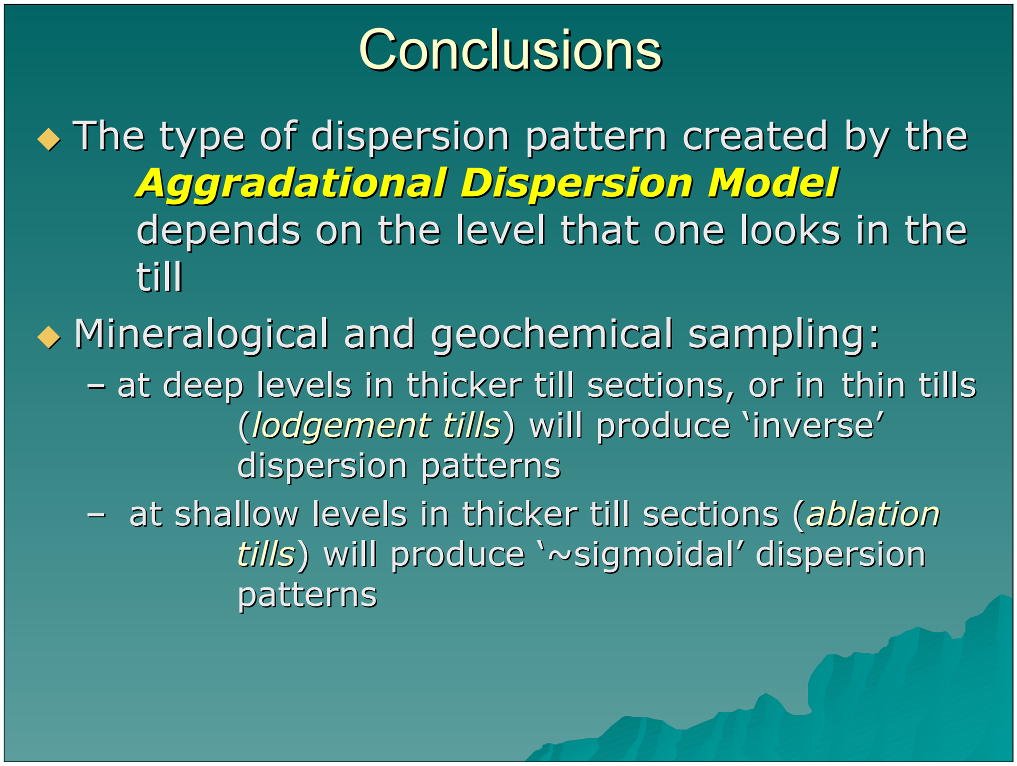
Now, no reports of till dispersion patterns similar to these simulated results have been made to date that might be used to test the Aggradational Dispersion Model; however, the number and detail of cross-sections through anomalous till dispersion trains is still rather limited. As a result, it remains to be seen whether similar dispersion characteristics and variations actually occur in nature.

Nevertheless, we now have a motivation to be on the lookout for such characteristics, and if anyone has observed such till dispersion patterns, I would love to hear from you!



In conclusion, the historical exponential and linear dispersion models are inconsistent with physical processes that erode, transport and deposit till.

An alternative Aggradational Dispersion model is consistent with these processes.



This model simultaneously explains both ‘exponential’-type and ‘linear’-type dispersion patterns via a single mechanism that is consistent with the locations where these patterns are observed (in lodgement and ablation till settings, respectively).

Conclusions

- The ***Aggradational Dispersion Model*** provides insight into how glacial dispersion patterns can be controlled by the '*erodability*' of the bedrock
  - hard rock => shallower dispersion
  - soft rock => steeper dispersion
  - soft anomaly => divergent dispersion
  - hard anomaly => convergent dispersion steepens

The model provides some insights into the variations that till dispersion trains might exhibit in nature. One possible factor that can control the dispersion train shape, size, dip, contacts, etc. is the background and anomalous rock erodability.

Future Work

- To date, the ***Aggradational Dispersion Model*** has been represented by a finite difference/material transfer model
- Need to develop a quantitative representation of the ***Aggradational Dispersion Model*** by solving this partial differential equation

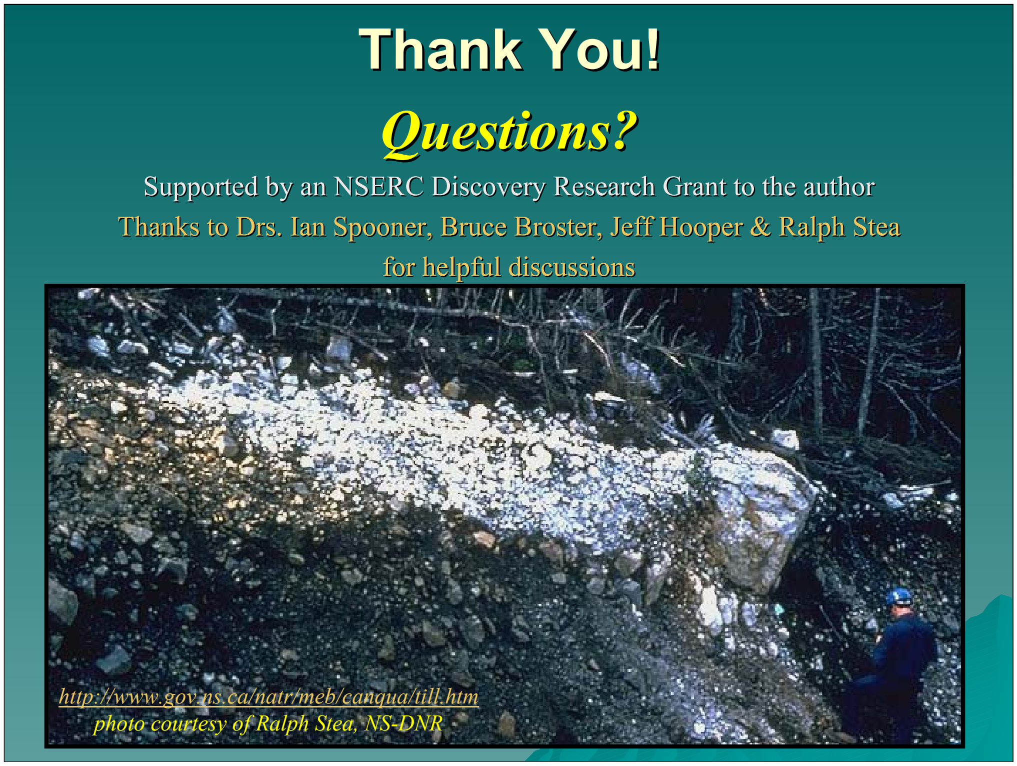
$$\left(\frac{\partial a}{\partial x}\right)+\left(\frac{\tau}{T}\right)\left(\frac{\partial a}{\partial z}\right)=k\nabla^{2}(a)=k\left(\frac{\partial^{2} a}{\partial x^{2}}+\frac{\partial^{2} a}{\partial z^{2}}\right)$$

- This will produce an equation describing the family of curves which can be regressed to estimate the location of an up-ice contact

Lastly, one should note that the Aggradational Dispersion Model has been formulated using a finite difference / material transfer template.

In order to be truly quantitative, an equation must be developed to allow fitting of a a real curve to till concentration data.

Future work involves the development of this equation via the solution of this partial differential equation. The solution to this PDE is a formula that will define a family of curves that match those observed in the various layers of the Aggradational Dispersion Model. This formula will ultimately be able to be regressed to observed till concentration data to allow calculation of the up-ice location of the source of any geochemical, mineralogical, or lithological anomaly.


Thank You!
Questions?
Supported by an NSERC Discovery Research Grant to the author
Thanks to Drs. Ian Spooner, Bruce Broster, Jeff Hooper & Ralph Stea
for helpful discussions
http://www.gov.ns.ca/natr/meb/canqua/till.htm
photo courtesy of Ralph Stea, NS-DNR

Thank You.